FEASIBILITY OF USING A TERNARY MODE FOR PULSE TORQUING A PENDULOUS ACCELEROMETER

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ABSTRACT

The feasibility of pulse torquing a pendulous accelerometer in a ternary mode is investigated. A comparison is made between basic ternary and binary systems. It is shown that a ternary system is basically more stable than a binary system in that stable oscillations are smaller. Thus errors in velocity indication are less for the ternary system.

The describing equations of a pulse torqued pendulous accelerometer are transformed into a non-dimensional form. By so doing, general conclusions are reached. Phase plane analysis is used to investigate the dynamic performance of the device. An error analysis based upon the phase plane analysis follows.

After conditions necessary for larger limit cycles are investigated, a method is devised for selecting system parameters such that larger limit cycles are prevented.

Finally, methods of compensation and error correction using switching logic are investigated. Representative switching logics are presented.

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CHAPTER 1

INTRODUCTION

1.1. HISTORY

A basic requirement of any inertial navigation system is that the system be able to obtain velocity. One common device developed to obtain velocity inertially, is the pendulous accelerometer. In such a device, a torque generator current required to keep a pendulum at null, is proportional to acceleration along a sensitive axis. When such current is integrated with respect to time, an analog of velocity is obtained.

With the addition of a digital computer to the inertial navigation system, a requirement is generated for an analog to digital converter. Such converters are inherently limited in accuracy of conversion.

As an alternative to providing a converter, a binary pulse torqued integrating accelerometer has been under study and development. Such an accelerometer has an output which consists of a train of pulses. These pulses may be fed directly to a digital computer. Each pulse represents an incremental velocity change along the sensitive axis of the accelerometer. By summing the train of pulses, taking into consideration their algebraic sign, a velocity indication is obtained.

In the operation of a binary pulse torqued integrating accelerometer, certain errors are introduced which are caused by inherent lag in the pendulum servo system.

1.2. A PENDULOUS ACCELEROMETER

In general, a pendulous accelerometer consists of a pendulous mass suspended in a viscous fluid (Figure 1-1). The pendulous mass is constrained to rotate about a single axis. An error detector is used to measure the deviation of the pendulous mass from a reference axis. After amplification, the error signal is used to command a torque generator in such a way that the pendulum is driven back toward its null position. The applied torque may, or may not be, a linear function of null angle.

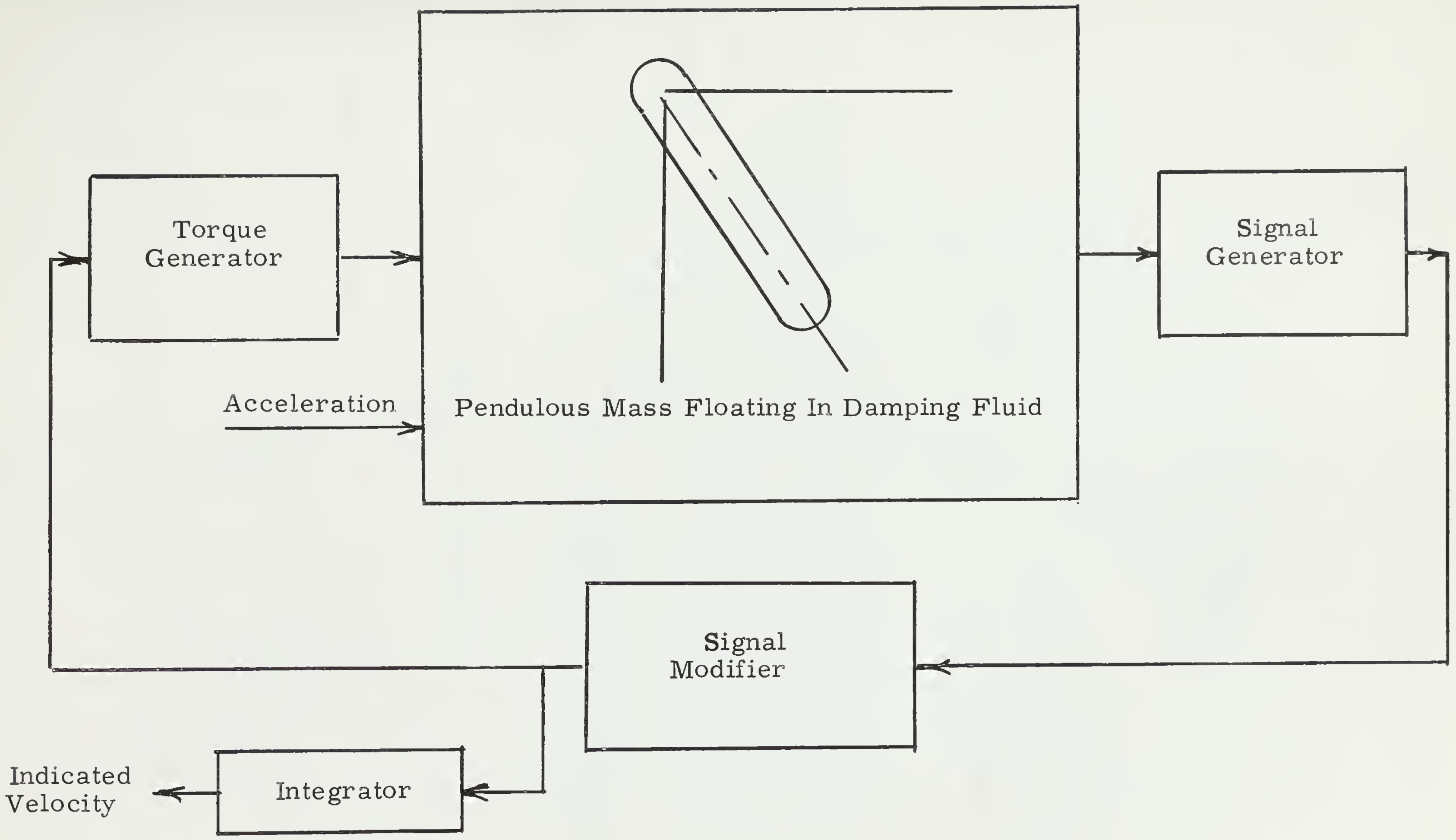


Figure 1-1. Pendulous Accelerometer.

The equation of motion of the pendulum is

$$J \frac{d^2A}{dt^2} + C \frac{dA}{dt} = M_{tg} - aP\cos A + a_{RA} P \sin A \tag{1.1}$$

where

P = Pendulosity of the pendulum

A = Angular position of the pendulum

J = Moment of inertia of the pendulum

C = Coefficient of viscous friction

M_{tg} = Torque Generator torque

a = Component of acceleration along the input axis of the accelerometer

a_{RA} = Component of acceleration along the Reference Axis of the accelerometer

t = Time.

For the purpose of this report, acceleration is defined as the second derivative of position with respect to time as viewed in a free-fall non-rotating coordinate system. This is not a restriction in that Equation (1.2) is valid in any coordinate system if the acceleration, a, is defined to include gravitational attraction and apparent accelerations associated with rotating frames.

The null angle, A, is maintained sufficiently small to justify small angle approximations being made. With this in mind, Equation (1.1) becomes:

$$J \frac{d^2A}{dt^2} + C \frac{dA}{dt} = M_{tg} - aP + a_{RA} PA \tag{1.2}$$

Rearranging and integrating:

$$v = \int_0^t a\,dt = \frac{1}{P} \int_0^t M_{tg}\,dt - \frac{J}{P}\frac{dA}{dt} - \frac{C}{P} A + \int_0^t a_{RA} A\,dt \tag{1.3}$$

where v is the velocity increase from time zero to time t. dA/dt and A are assumed zero at time equal to zero.

By definition:

$$v_i = \frac{1}{P} \int_0^t M_{tg}\, dt = \text{indicated velocity} \tag{1.4}$$

$$e_n = \frac{J}{P} \frac{dA}{dt} + \frac{C}{P} A = \text{null error} \tag{1.5}$$

$$e_c = - \int_0^t a_{RA} A\, dt = \text{cross-coupling error} \tag{1.6}$$

$$e = e_n + e_c = \text{total error.} \tag{1.7}$$

Substituting (1.5) and (1.6) into (1.3):

$$v = \frac{1}{P} \int_0^t M_{tg}\, dt - e_n - e_c. \tag{1.8}$$

1.3. A BINARY PULSED INTEGRATING ACCELEROMETER

Figure 1-2 is a block diagram of a binary torqued integrating accelerometer. The pendulum is always torqued by a current of constant magnitude. The sense or direction of the current through the torque generator may be changed, however. The error detector is sampled at a rate synchronous with a computer clock. At each sample instant, the current to the torque generator is switched or not switched as necessary to drive the pendulum toward null. Lag is introduced into the restoring system by the use of a discrete time between samples.

The state of the applied torque is based upon the error signal at a sample instant. Practical error detectors have a finite insensitive zone near their null position. If at a sample instant, the pendulum is in the insensitive zone (hereafter called the dead zone), the applied torque is maintained in a sense based upon the last available information. Thus additional lag is introduced.

As a result of lag in the error channel, the pendulum oscillates in stable modes or limit cycles. These limit cycles if large allow the null error, e_n, to be several velocity increments in magnitude.

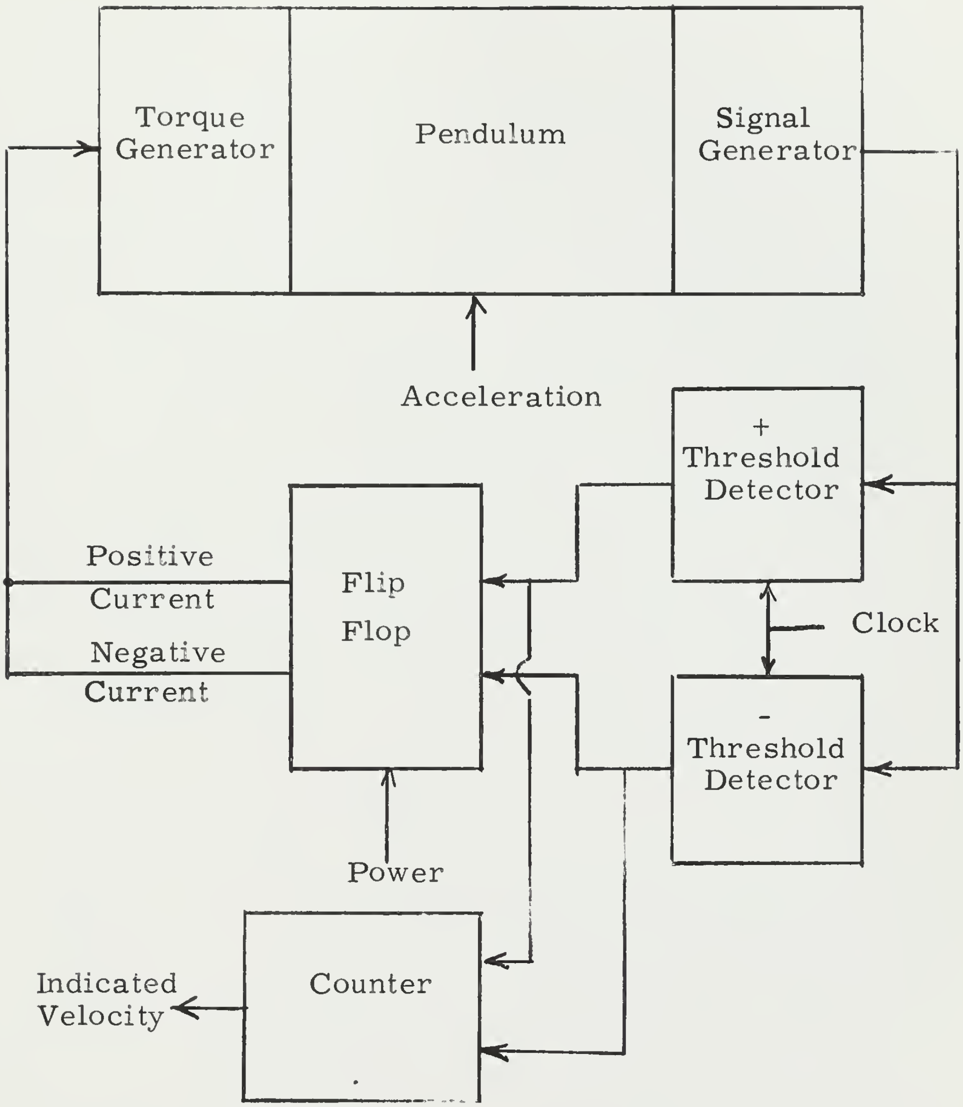


Figure 1-2. A Binary Pulsed Pendulous Integrating Accelerometer.

The chapters to follow investigate the feasibility of reducing the errors in a pulse torqued integrating accelerometer by torquing the pendulum in a ternary mode.

CHAPTER 2

A TERNARY PULSED INTEGRATING ACCELEROMETER

2.1. GENERAL

The size of the limit cycle of a pulsed accelerometer may be reduced by one of two methods. First, the lag which causes the excessive limit cycles may be reduced. Second, lead may be introduced to compensate for lag already in the system.

If, in a pulse torqued accelerometer, the pendulum null angle is such that the acceleration reaction torque is in a direction to reduce the null angle, it can be anticipated that the null angle will eventually be driven to zero without the use of additional torque. Thus, the use of applied torque to aid acceleration reaction, coupled with the discrete time increments at which it must be applied, introduces unnecessary lag to the system. This elementary observation prompted this investigation.

If torquing to aid acceleration reaction torque is to be prevented, a "no torque" mode of operation must be introduced. Since both positive and negative accelerations are to be expected, positive and negative torque modes must be retained. Thus a ternary mode of operation is suggested.

2.2. COMPUTER CONSIDERATIONS

The computer most usually used in inertial guidance systems is the Digital Differential Analyzer (DDA). The Digital Differential Analyzer can be mechanized to handle binary or ternary inputs. The ternary scheme is more accurate in that it has less round-off errors, but requires more equipment to realize.

The DDA must accept information at a clock rate. Thus the requirement that the torque pulses be synchronous with a computer clock will be maintained in a ternary system.

2.3. INFORMATION AVAILABLE

The accuracy of a pendulous accelerometer depends upon the deviation of the pendulum from its null position being maintained small. With such small deviations allowed, pendulum velocity is extremely hard to

measure through system noise. For this reason, it is considered that pendulum velocity is not directly available.

Pendulum position is available if the pendulum is not in the error detector dead zone. If the pendulum is in the dead zone, the fact that it is in the dead zone is known. This fact may be used in switching logic.

The past history of applied torque is available and may be useful in that it represents the past motions of the pendulum. Since input acceleration is a slow changing function with respect to pendulum motion, the past history of pendulum motion might be useful in predicting future motion. Thus, lead might be introduced.

2.4. POWER SUPPLY CONSIDERATIONS

If torque pulses are to accurately indicate the same velocity quantum with repeatibility, the torque generator current must be maintained extremely constant. The current taken from the power supply is constant in a binary system. A ternary system requires that the current be switched on and off. Because of the variation in current taken from the power supply, a better power supply would be required for the ternary system.

An alternative to switching the power supply off might be to switch it to a dummy load. At any rate a ternary system would require more or better power supply equipment.

2.5. A BASIC TERNARY PULSED INTEGRATING ACCELEROMETER

Figure 2-1 shows a basic ternary pulsed integrating accelerometer. Switching logic is based upon pendulum position. If at a sample instant (clock pulse), the pendulum is not in the error detector dead zone, the pendulum is torqued for the next sample interval, in a direction toward null. If the pendulum is in the dead zone at a sample instant, the pendulum is not torqued for the next sample period.

For a pulsed system, Equation (1.8) can be written:

$$v = \frac{1}{P} \sum_{i=0}^{n} \delta_i M t_s - e_n - e_c - e_d \qquad (2.1)$$

where

t_s = time between samples

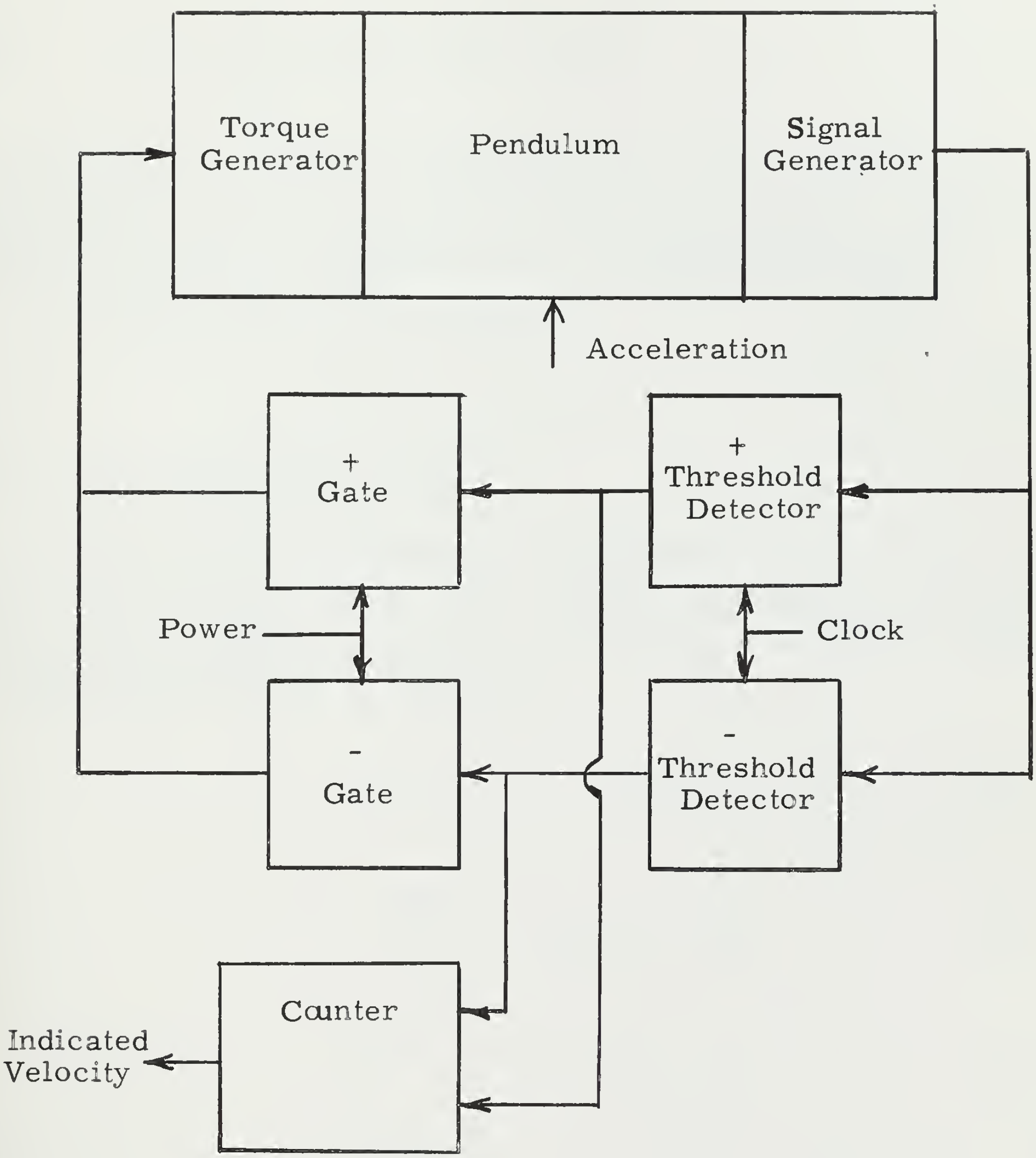


Figure 2-1. A Ternary Pulsed Pendulous Integrating Accelerometer.

$$\delta_i = \begin{cases} +1 & \text{when torquing in a positive direction} \\ 0 & \text{when not torquing} \\ -1 & \text{when torquing in a negative direction} \end{cases}$$

and
$$e_d = \frac{1}{P} \sum_{i=0}^{n} \delta_i \, M \, t_s - \frac{1}{P} \int_0^t M \, dt .$$

The digital error is described by Figure 2-2. It is caused by an instantaneous addition of velocity quantum in the computer. The actual torque-time pulse has taken place over a finite time interval.

The total error is now defined by:

$$e = e_n + e_c + e_d \tag{2.2}$$

2.6. NON-DIMENSIONAL FORM

It is desirable to express the equations of motion as free of as many system constants as possible. By so doing, general conclusions can be easier reached. By proper substitutions of relations in Table 2-I into Equations (1.2), (2.1), and (2.2):

$$\ddot{\phi} + \dot{\phi} = (\delta - r + r_{RA}\, \phi) \tag{2.3}$$

$$V = \sum_{i=0}^{n} \delta_i - E_n - E_c - E_d \tag{2.4}$$

$$E = E_n + E_c + E_d \tag{2.5}$$

where
$$\dot{\phi} \triangleq \frac{d\phi}{d\tau} .$$

Two system parameters must be specified before an analysis of system performance can be made. Equation (2.3) will have to be piecewise integrated between samples. Therefore T_s, the ratio of sample time to pendulum time constant, must be specified. The non-dimensional

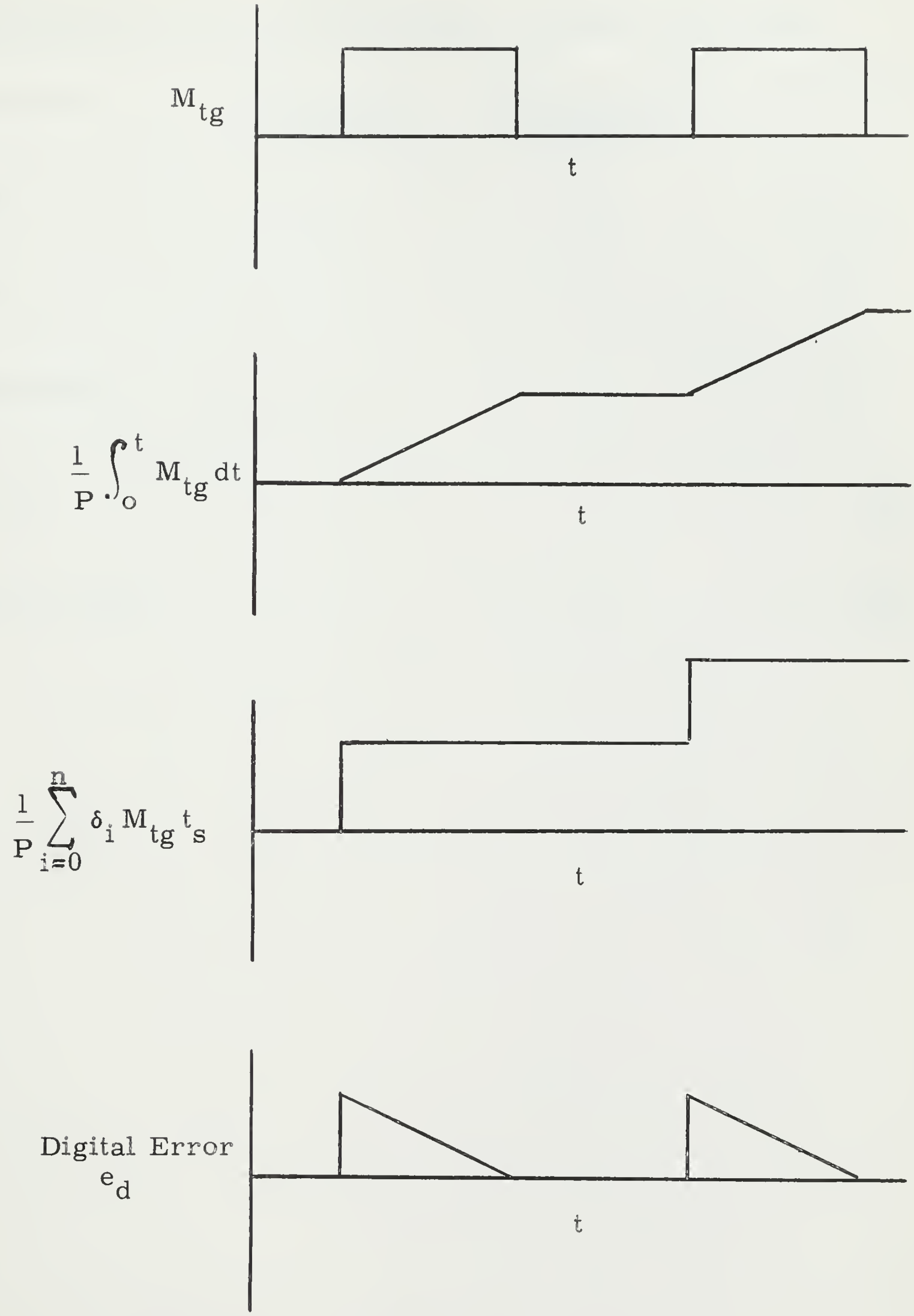


Figure 2-2. Example of Digital Error.

TABLE 2-I

DIMENSIONAL TO NON-DIMENSIONAL TRANSFORMATIONS

Parameter	Dimensional Notation	Non-Dimensional Notation	Relation
Angle	A	Φ	$\Phi = \frac{AC}{MJ}$
Time	t	τ	$\tau = \frac{Ct}{J}$
Acceleration	a	r	$r = \frac{aP}{M}$
Velocity	v	V	$V = v \frac{PC}{MJ}$
Velocity Error	e	E	$E = e \frac{PC}{MJ}$

dead zone, ϕ_Δ, must be specified in order to tell where switching will take place.

2.7. PHASE PLANE ANALYSIS

In general, the acceleration is essentially constant when viewed over several sample intervals. Thus, acceleration is considered constant in this analysis. For the purpose of analysis, r is assumed much greater than $r_{RA}\,\phi$ in prescribing the motion of the pendulum. This does not imply that the cross-coupling error is negligible but rather that the dynamic behavior of the pendulum is not greatly affected by the cross-coupling acceleration torque. This type analysis cannot be made for zero acceleration along the sensitive axis in that r_{RA} is the only forcing function when the pendulum is not in a torque mode.

With the $r \gg r_{RA}\,\phi$ approximation, Equation (2.3) becomes:

$$\ddot{\phi} + \dot{\phi} = (\delta - r) \qquad (2.6)$$

Since δ remains constant between torque switches, Equation (2.6) may be integrated as a first order linear equation in $\dot{\phi}$, with results being valid between torque switches:

$$\dot{\phi} = \dot{\phi}_o\, e^{-\tau} + (\delta - r)(1 - e^{-\tau}) \qquad (2.7)$$

where $\dot{\phi}_o$ is the pendulum velocity at $\tau = 0$.

$$\phi = \phi_o + \dot{\phi}_o\,(1 - e^{-\tau}) + (\delta - r)(\tau + e^{-\tau} - 1) \qquad (2.8)$$

where ϕ_o is the null angle at $\tau = 0$.

It is believed that a phase plane ($\dot{\phi}$ versus ϕ plot) study lends the best understanding of second order nonlinear equations. It is assumed that the reader has a basic understanding of phase plane analysis. Since an elastic restraint term is absent from Equation (2.6), a phase plane template can be constructed for a given value of $(\delta - r)$. Such a template can be used over and over again by sliding it along the ϕ axis. This is possible because the phase trajectory is independent of the relative

position of the $\phi = 0$ axis. Time must be plotted as a parameter along the trajectory in order to tell when switching will take place.

Since the null error has been defined as the sum of angle and angle rate, it can be found for any point on the phase plane by projecting the point in question onto an axis which is perpendicular to all $\dot{\phi} + \phi$ = constant lines.

Adding Equations (2.7) and (2.8):

$$E_n = \phi_o + \dot{\phi}_o + (\delta - r)\,\tau. \qquad (2.9)$$

Equation (2.9) states that the null error is a linear function of time between torque switches.

2.8. PREVENTING APPLIED TORQUE FROM AIDING ACCELERATION REACTION TORQUE

It is desirable that torque not be applied in a direction that will aid the acceleration reaction torque. Appendix B shows that if the non-dimensional dead zone, ϕ_Δ, is greater than the non-dimensional sample time, T_s, a limit cycle cannot exist for zero input acceleration. In general, if ϕ_Δ is greater than T_s, the pendulum in steady state, will remain out of the switch zone that will switch torque to aid acceleration reaction (Figures 2-3 through 2-12). For accelerations greater than zero, the forcing function of Equation (2.6) tends to bend the phase trajectory away from the undesirable zone.

2.9. EXAMPLE

In any analysis of a pulsed accelerometer, the non-dimensional sample time and the non-dimensional dead zone are functions of system parameters. For an illustrative example of analysis by phase plane methods, the non-dimensional sample time and the non-dimensional dead zone are both taken as unity. The non-dimensional dead zone is assumed greater than the non-dimensional sample time by an infinitesimal amount, such that limit cycles cannot exist for zero acceleration.

Figures 2-3 through 2-12 are phase plane plots of the system response for various values of non-dimensional acceleration. For magnitudes of non-dimensional acceleration equal to or greater than one, the

restoring torque cannot overcome the acceleration reaction torque. The system becomes unconditionally unstable under such conditions. For accelerations within the designed range of the accelerometer, the non-dimensional acceleration will never have a magnitude greater than one.

Phase plane plots of a binary torqued system are shown in Figure 2-3 through 2-12 for comparison purposes. The binary and ternary systems have identical system parameters. As could be expected, the limit cycles of the ternary system are smaller than those of the binary system for the same input accelerations. Higher order limit cycles exist for both systems. Such limit cycles give rise to errors in velocity indication. For the associated accelerations, all possible limit cycles have been shown in Figures 2-3 through 2-12. The conditions under which higher order limit cycles may exist in a ternary system are investigated in Chapter 3 and Appendix C.

2.10. ERROR ANALYSIS

From phase plane studies, it is noted that limit cycles for a given input acceleration have a certain freedom in position about the switching line. The exact amount of freedom that a simple limit cycle possesses has been determined and plotted as a function of T_s in Appendix C.

Figures 2-13 through 2-22 are plots of null error versus time for the phase plane plots of Figures 2-3 through 2-12. Again, a plot is shown for a binary system for comparison. The error plots are not unique in that the initial conditions determine the exact location of the limit cycle. The null error is bounded by a region determined by the freedom in position of limit cycle, ϕ_f, plotted in Appendix C.

It is noted that the peak null error is always less for the ternary system, usually by as much as one to two velocity counts.

The cross-coupling error is equal to the product of the component of acceleration along the reference axis, and the average value of the null angle. Both systems are biased in that the average null angle is not zero for accelerations greater than zero. In general, the average null angle is greater for the ternary system if the acceleration along the input axis is small and is greater for the binary system if the acceleration along the input axis is large. The bias in null angle is equivalent to the forced

dynamic error in a pendulous accelerometer with linear feedback.

The maximum magnitude of the average null angle is always less than the magnitude of the dead zone. The maximum value of the cross-coupling error is

$$\left| E_c \right|_{max} \leqq r_{RA}\, \phi_\Delta \tag{2.10}$$

In general, the digital error, E_d, is of opposite sign than the null error, thereby reducing the total error. The digital error is the error introduced by an incremental summation being made in the counter, instead of a true integration of torque.

The error equations used in this analysis have assumed that the pendulum is at rest in its null position when the problem commences. An exact analysis will require that the error associated with initial conditions be subtracted from the final error to determine the actual error in a particular problem.

2.11. REDUCTION IN ERRORS

It is apparent that errors could be reduced if the size of the limit cycle is reduced. Two approaches will be taken to this problem. First, system parameters will be adjusted in an attempt to minimize the limit cycle. Second, a system of compensation by switching logic will be investigated.

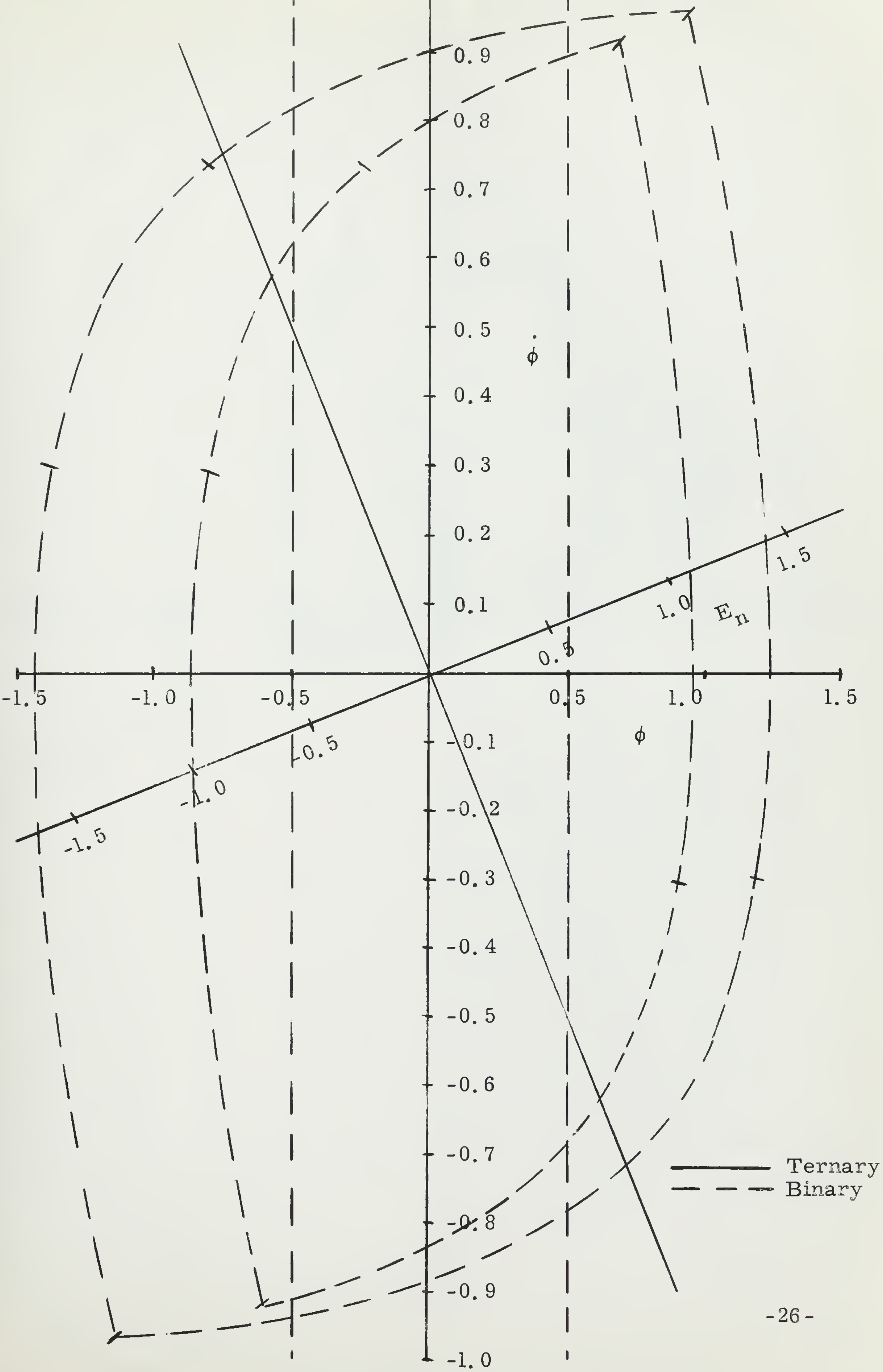


Figure 2-3. Steady State Limit Cycle: $T_s = 1$, $r = 0$.

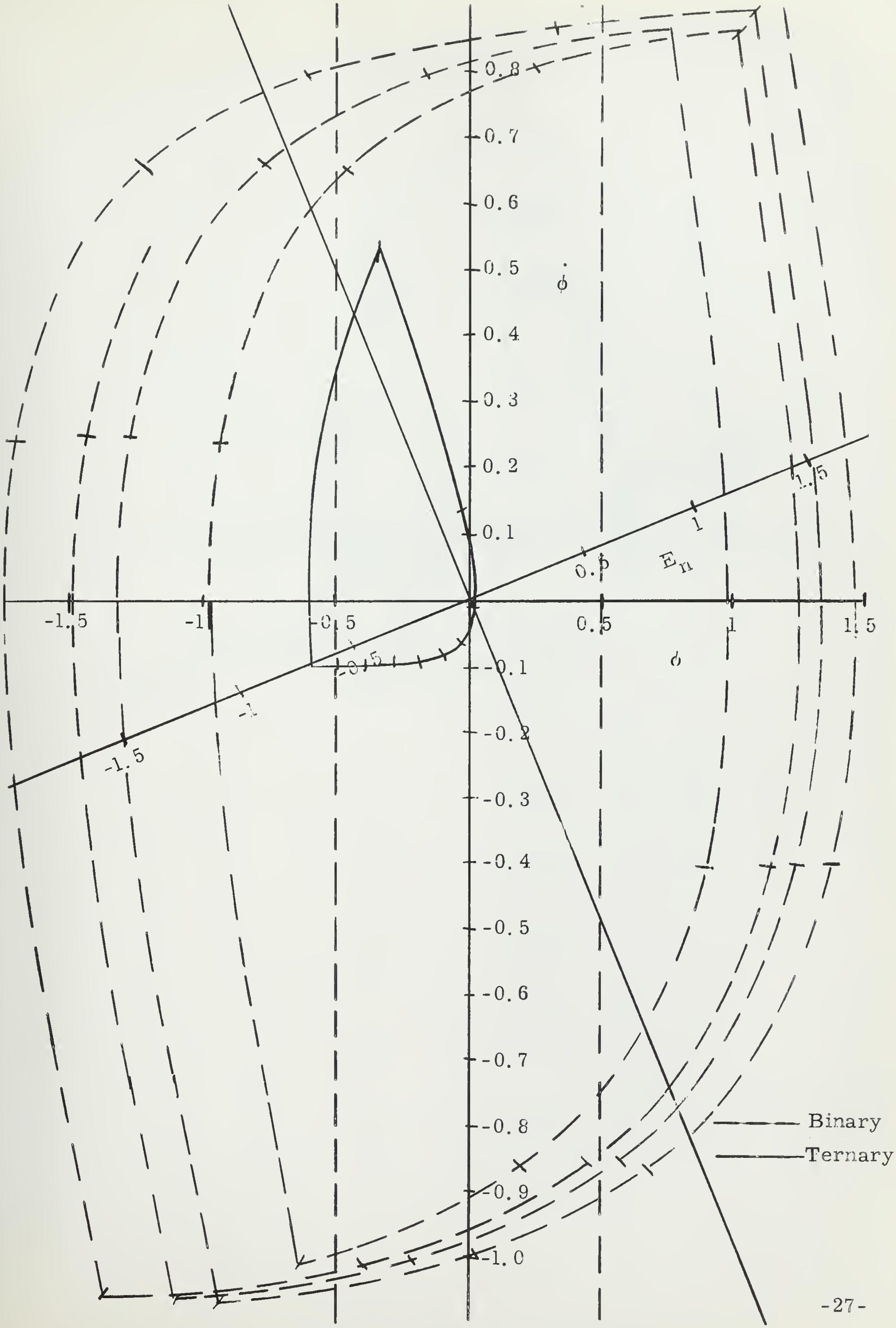


Figure 2-4. Steady-State Limit Cycle: $T_s = 1$, $r = 0.1$.

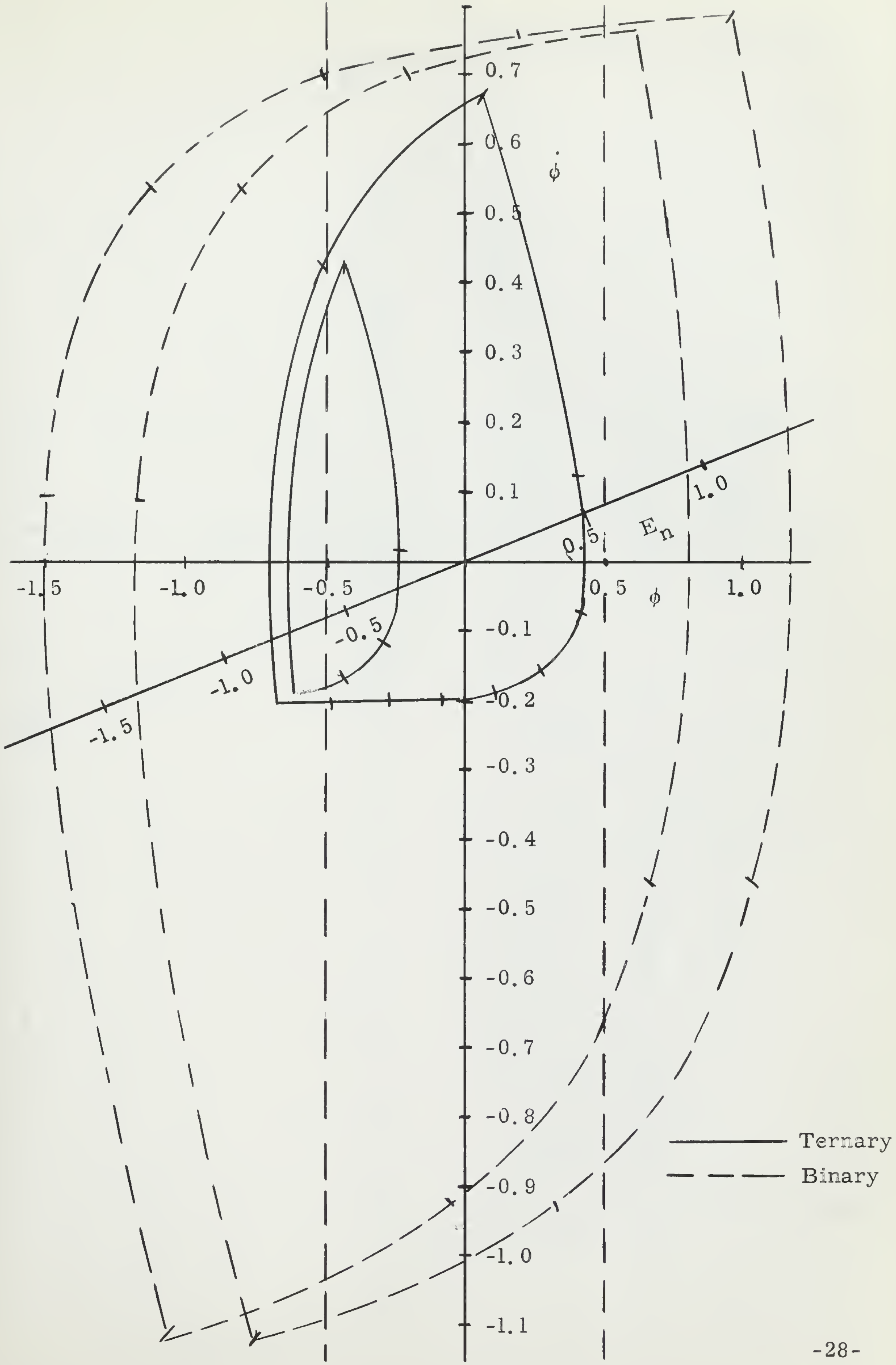


Figure 2-5. Steady State Limit Cycle: $T_s = 1$, $r = 0.2$.

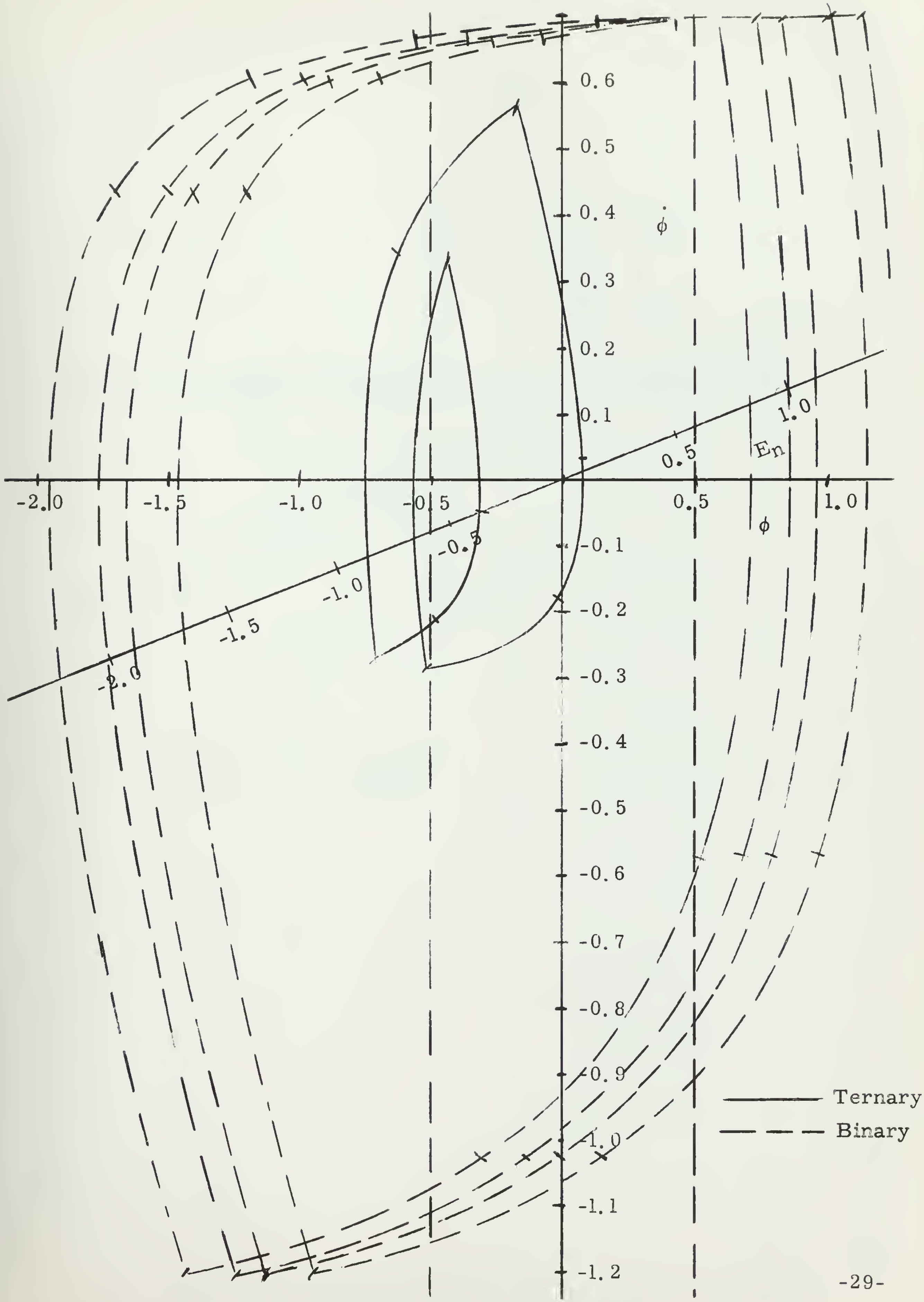


Figure 2-6. Steady State Limit Cycle: $T_s = 1$, $r = 0.3$.

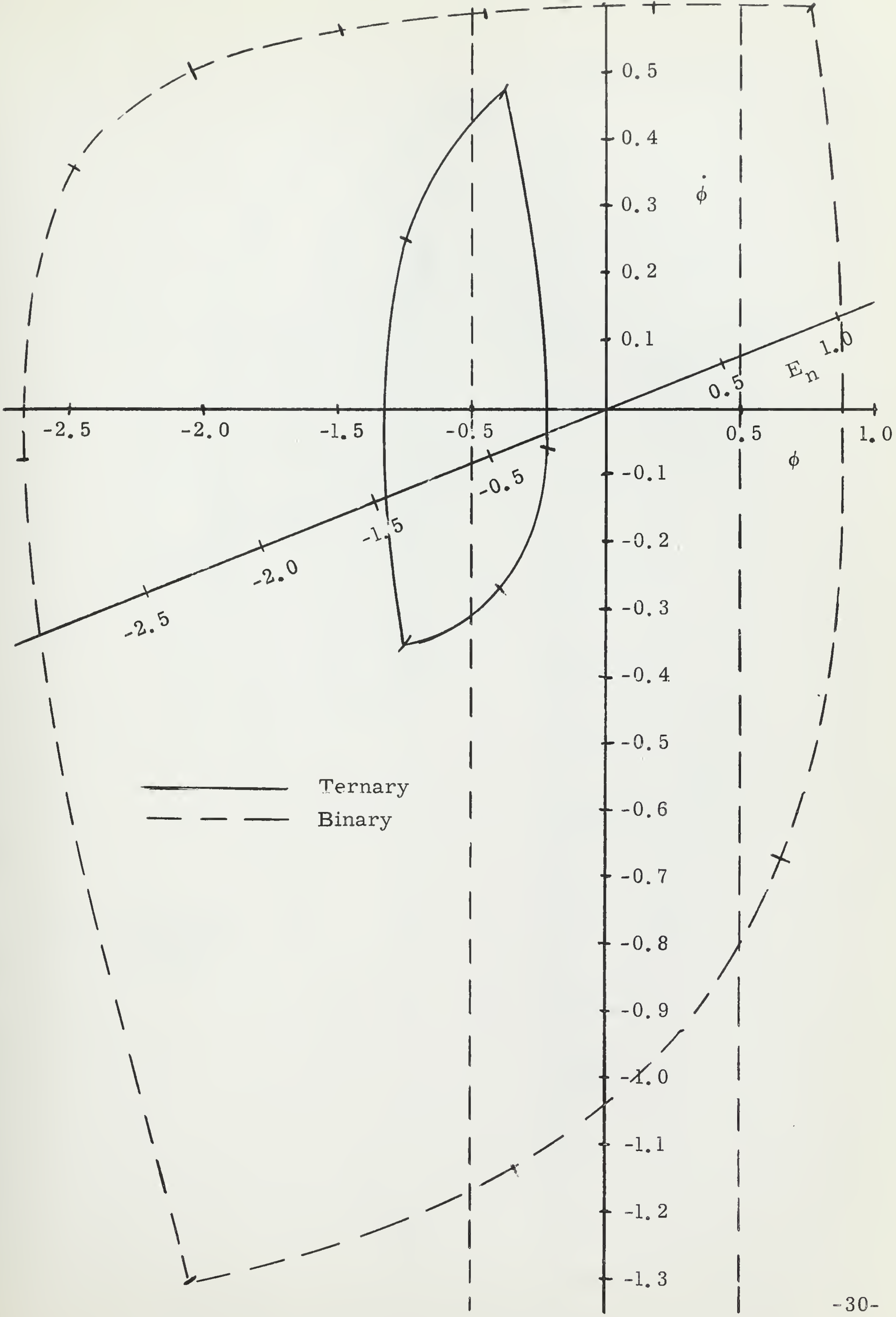


Figure 2-7. Steady State Limit Cycle: $T_s = 1$, $r = 0.4$.

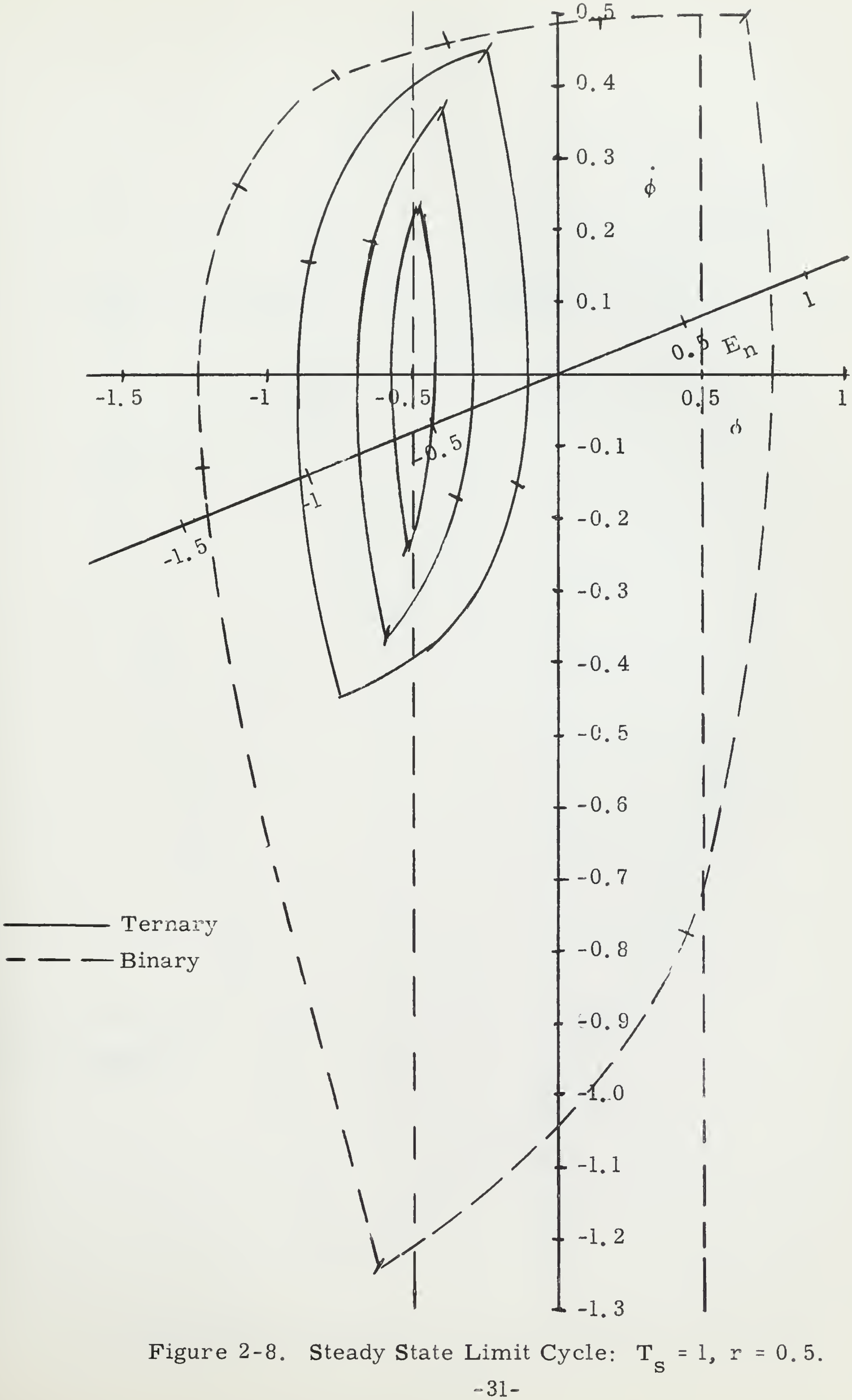


Figure 2-8. Steady State Limit Cycle: $T_s = 1$, $r = 0.5$.

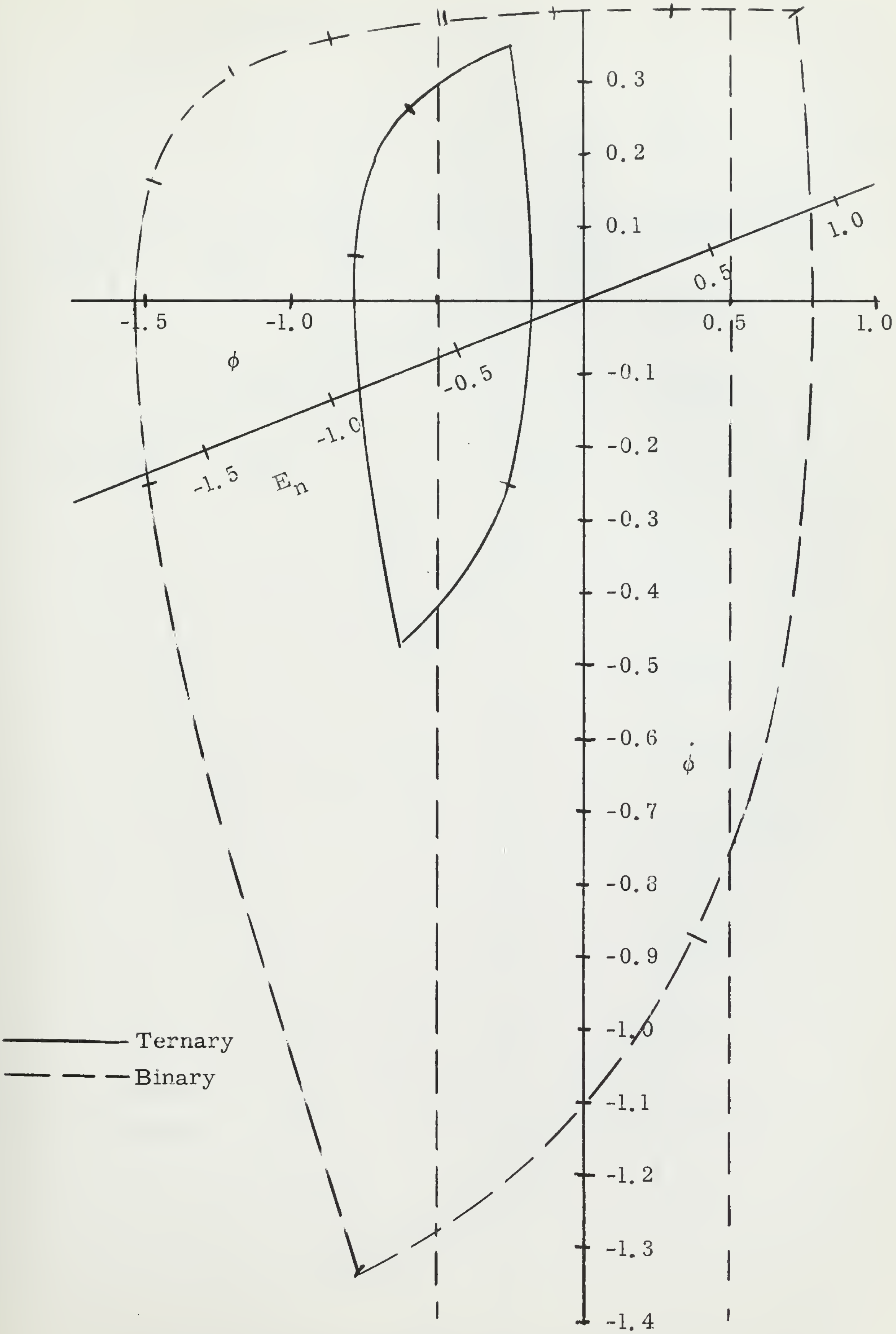


Figure 2-9. Steady State Limit Cycle: $T_s = 1$, $r = 0.6$.

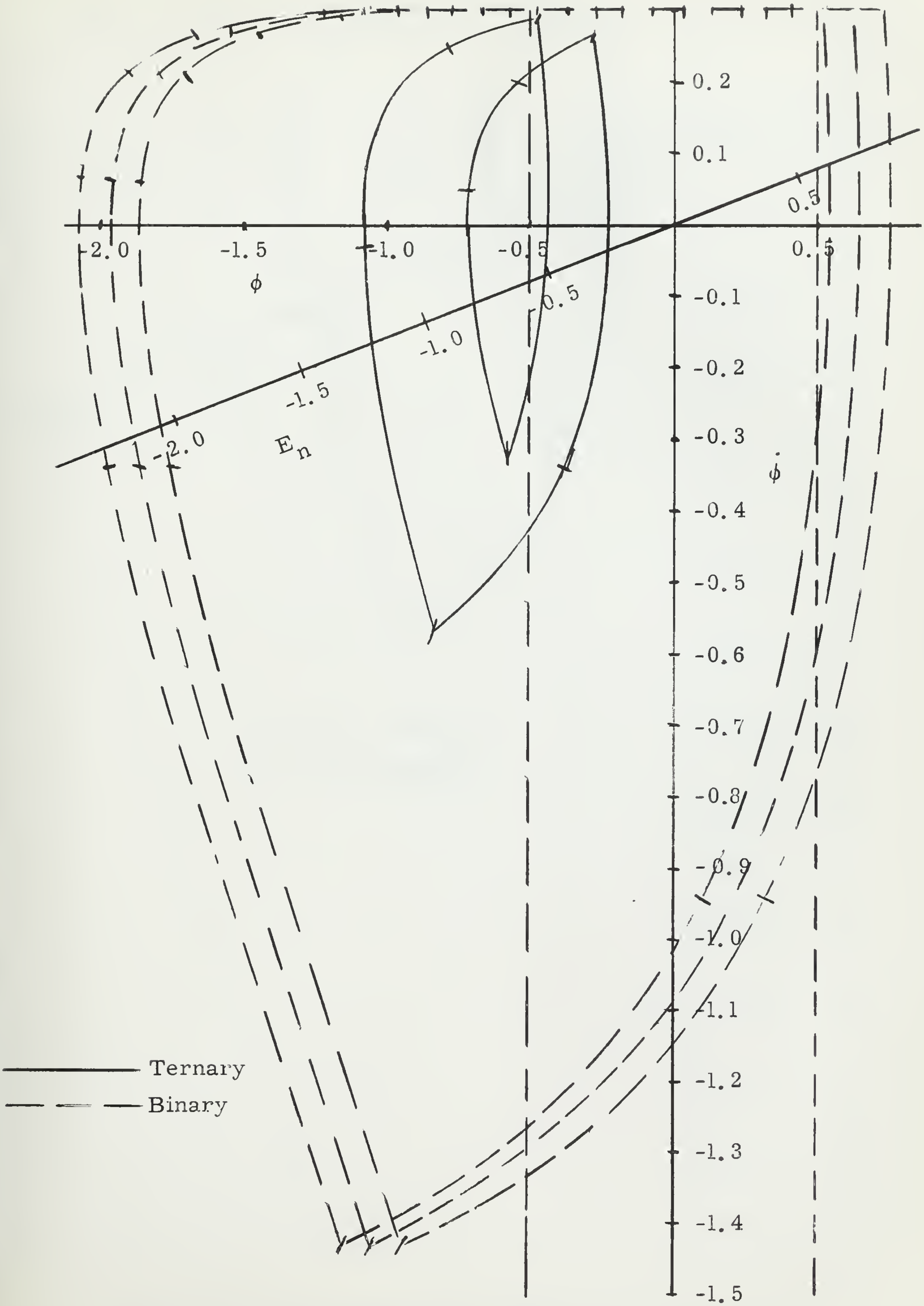


Figure 2-10. Steady State Limit Cycle: $T_s = 1$, $r = 0.7$.

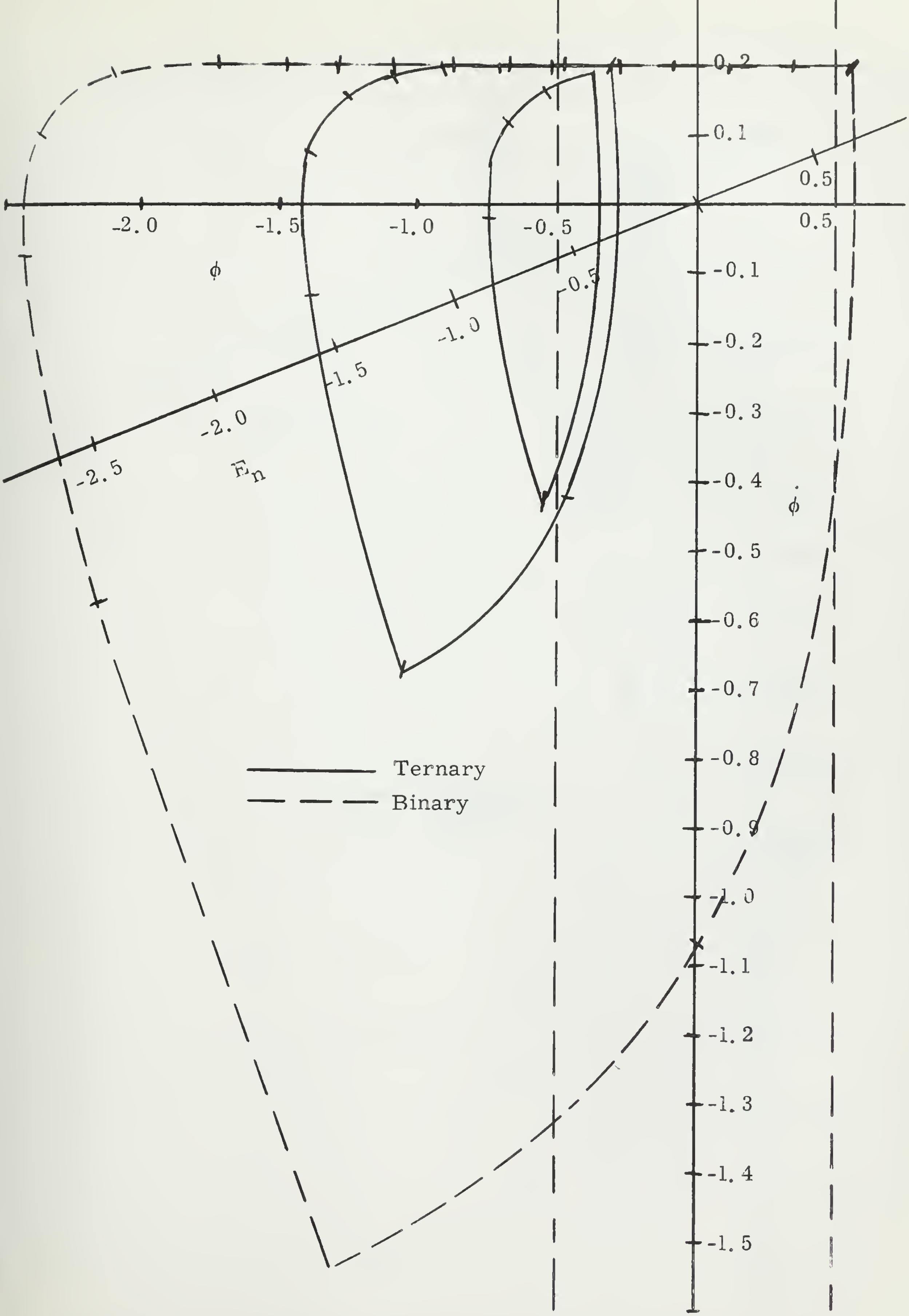


Fig. 2-11. Steady State Limit Cycle: $T_s = 1$, $r = 0.8$.

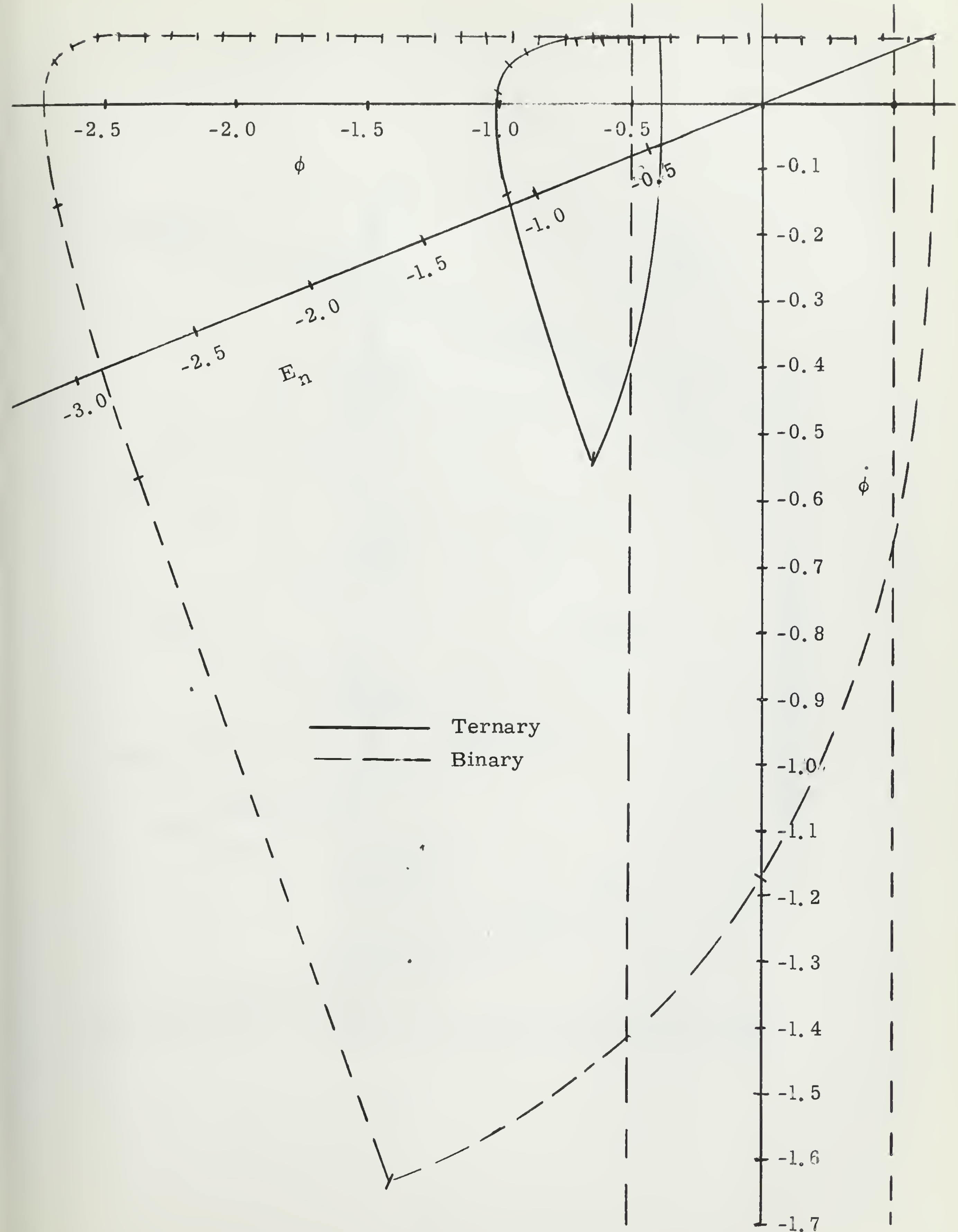


Figure 2-12. Steady State Limit Cycle: $T_s = 1$, $r = 0.9$.

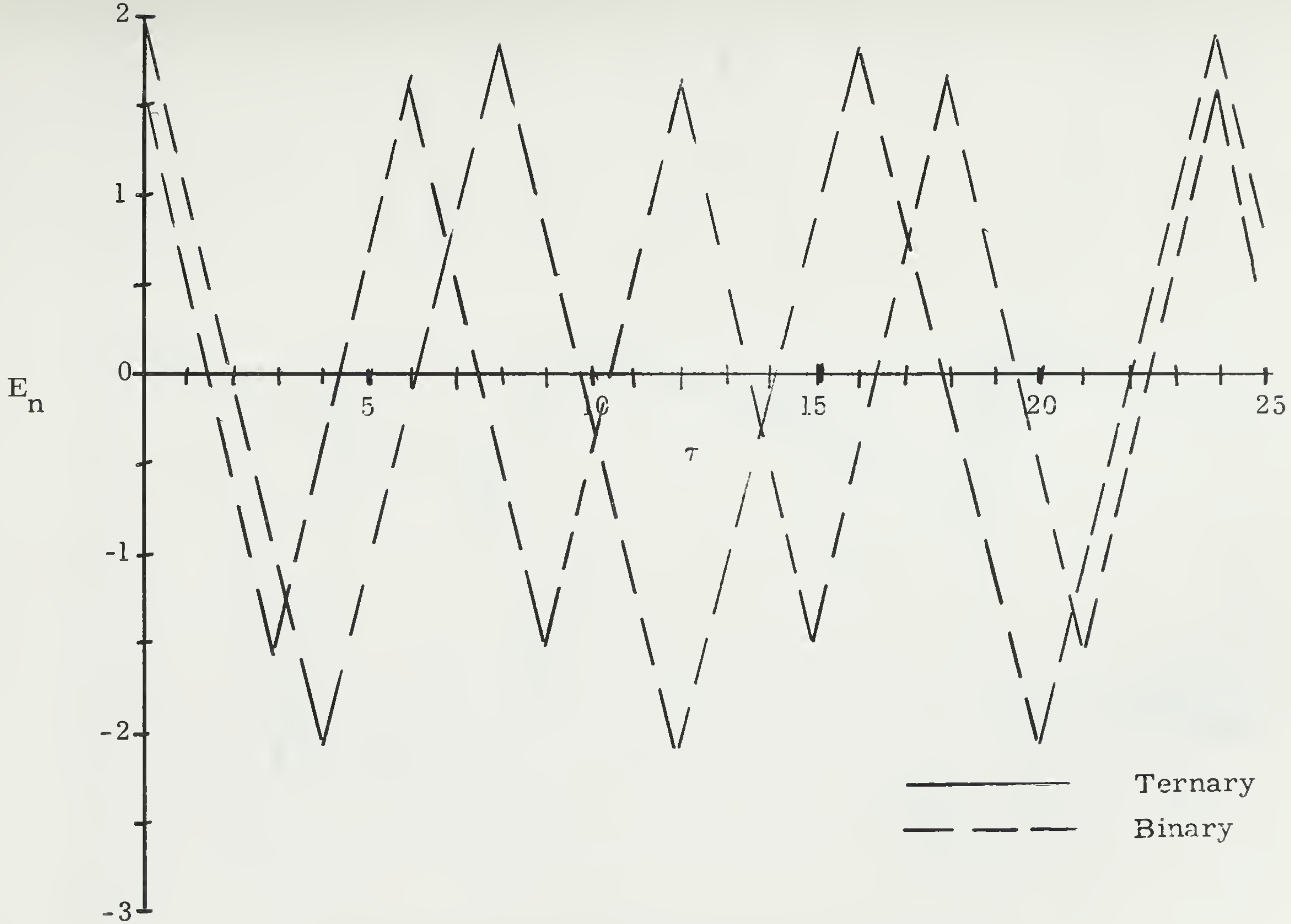


Figure 2-13. Null Error: $T_s = 1$, $r = 0$.

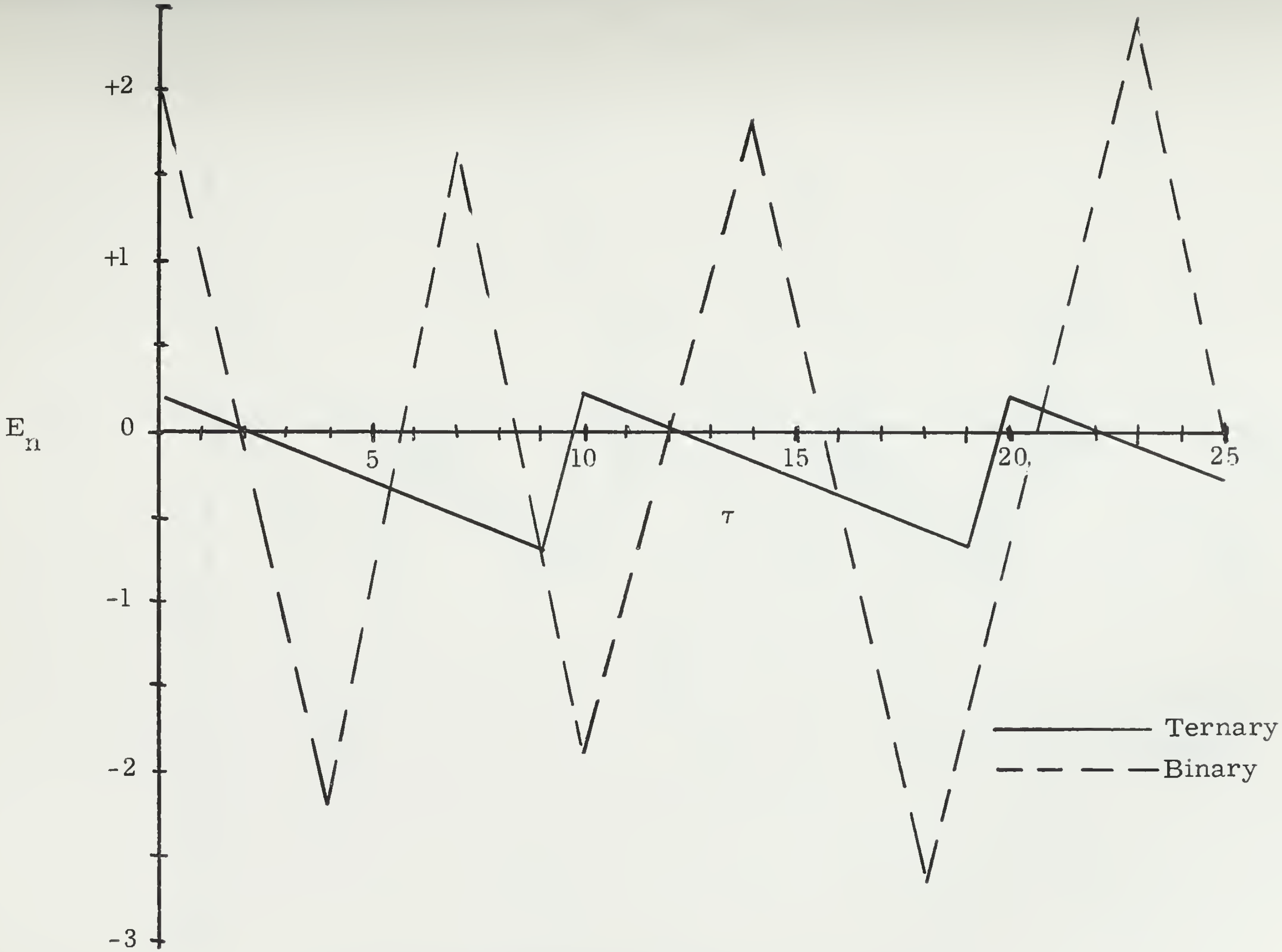


Figure 2-14. Null Error: $T_s = 1$, $r = 0.1$

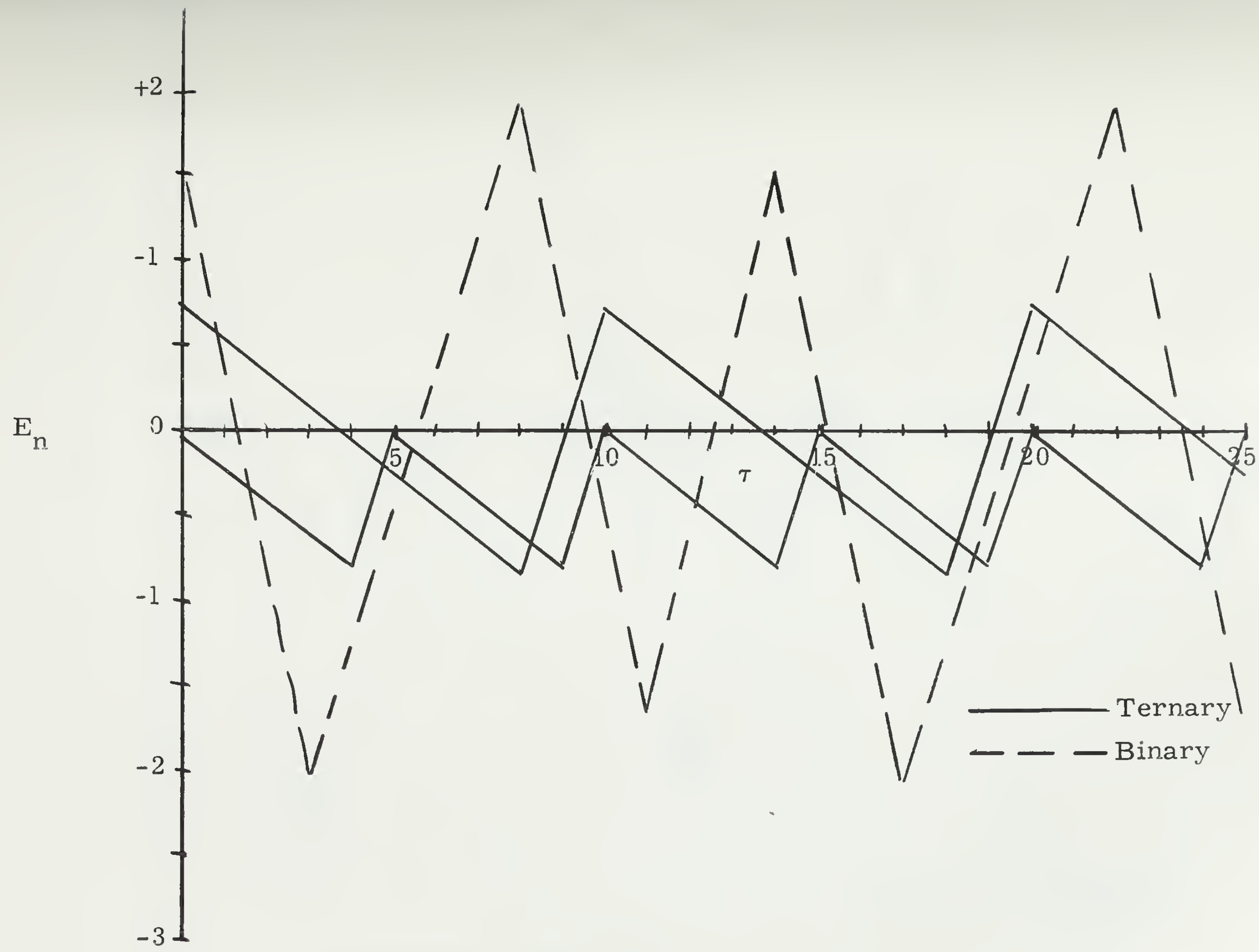


Figure 2-15. Null Error: $T_s = 1$, $r = 0.2$.

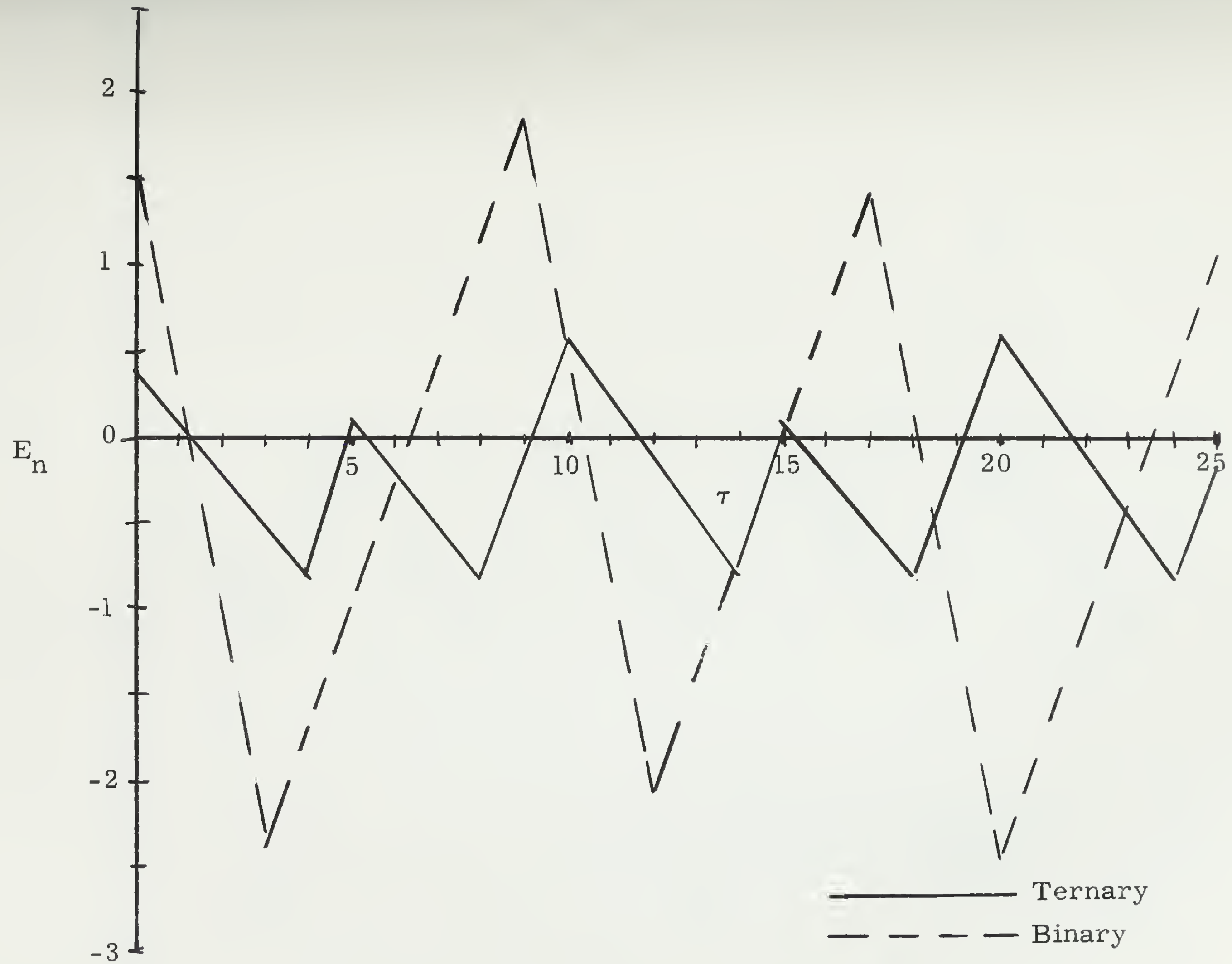


Figure 2-16. Null Error: $T_s = 1$, $r = 0.3$.

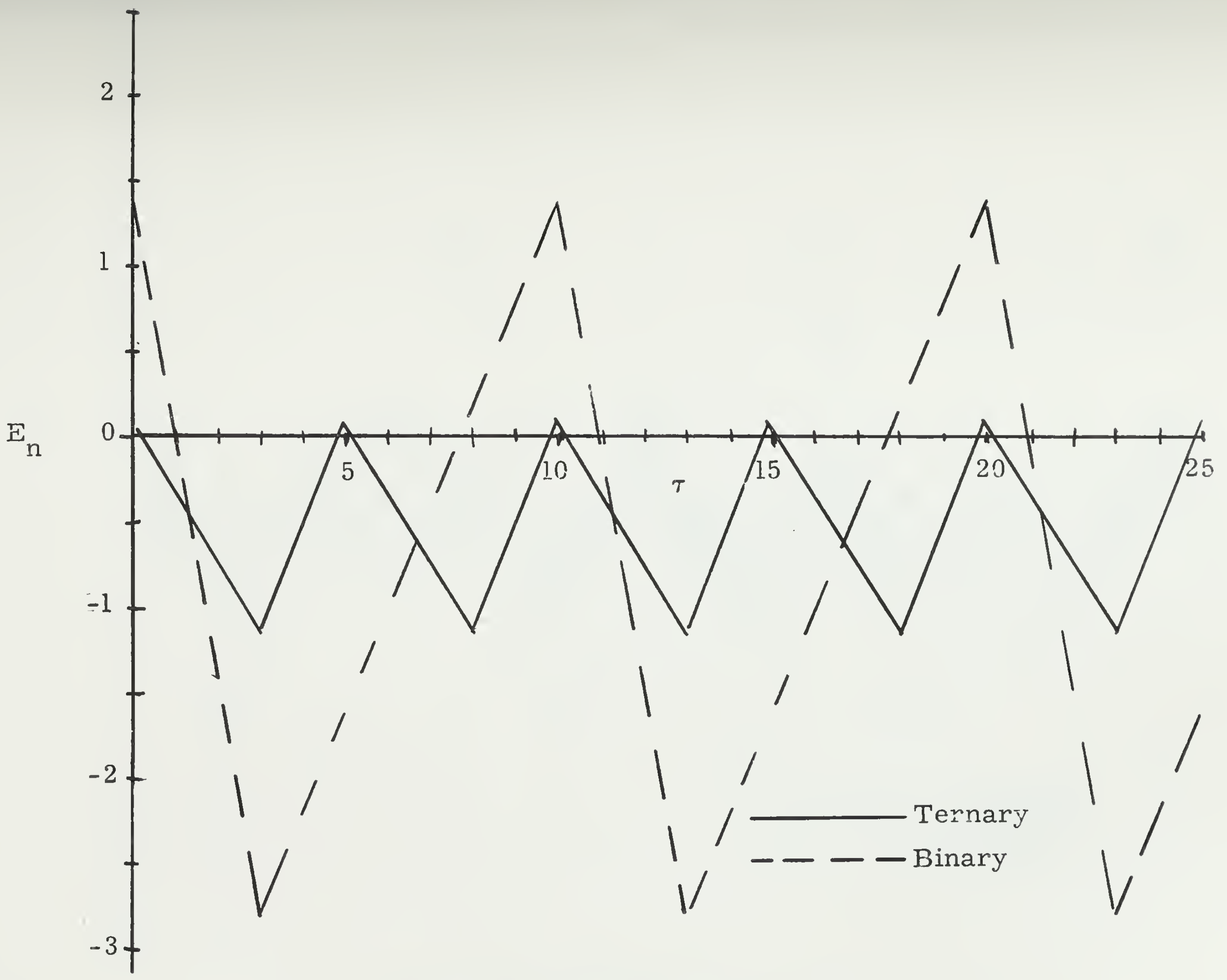


Figure 2-17. Null Error: $T_s = 1$, $r = 0.4$.

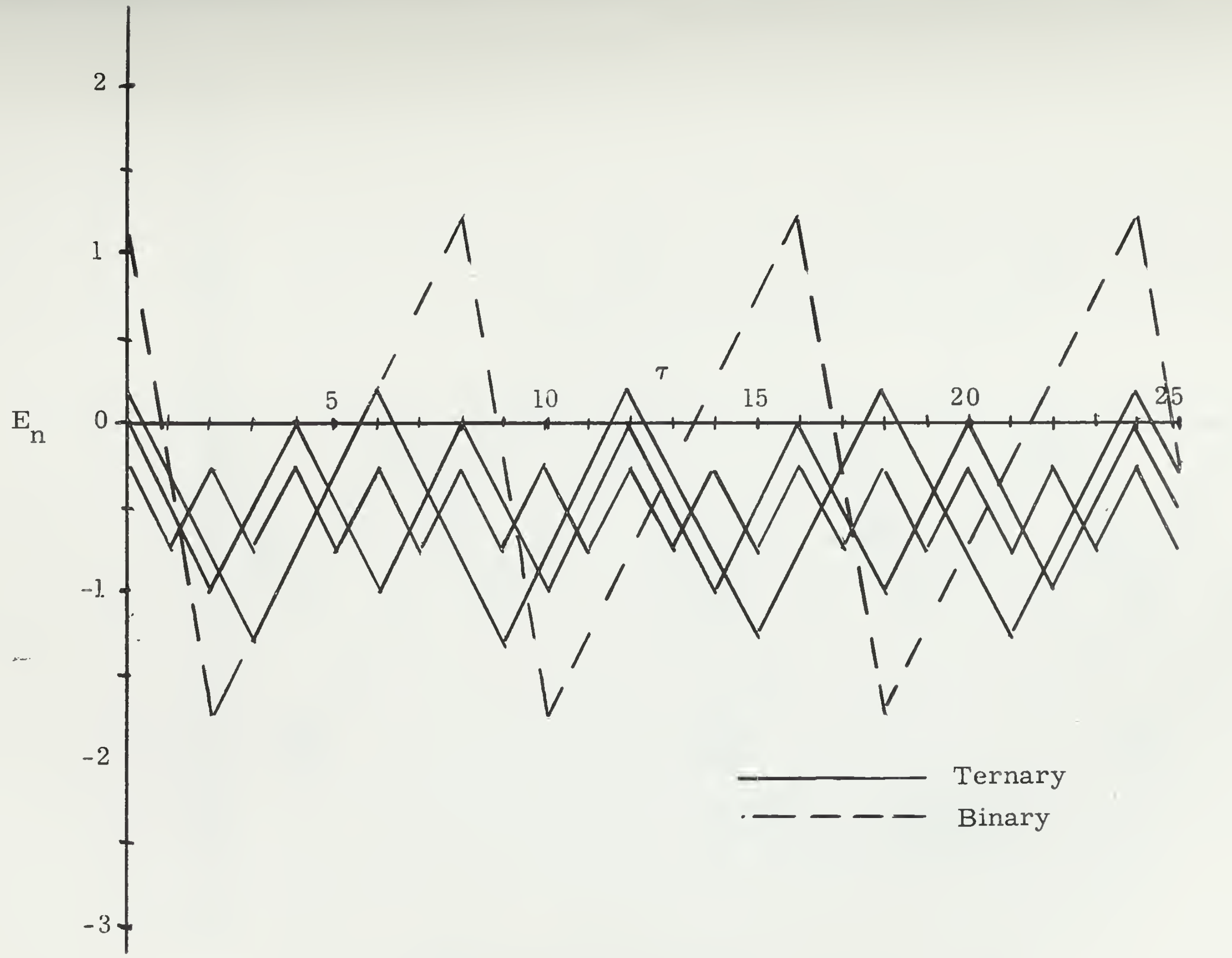


Figure 2-18. Null Error: $T_s = 1$, $r = 0.5$.

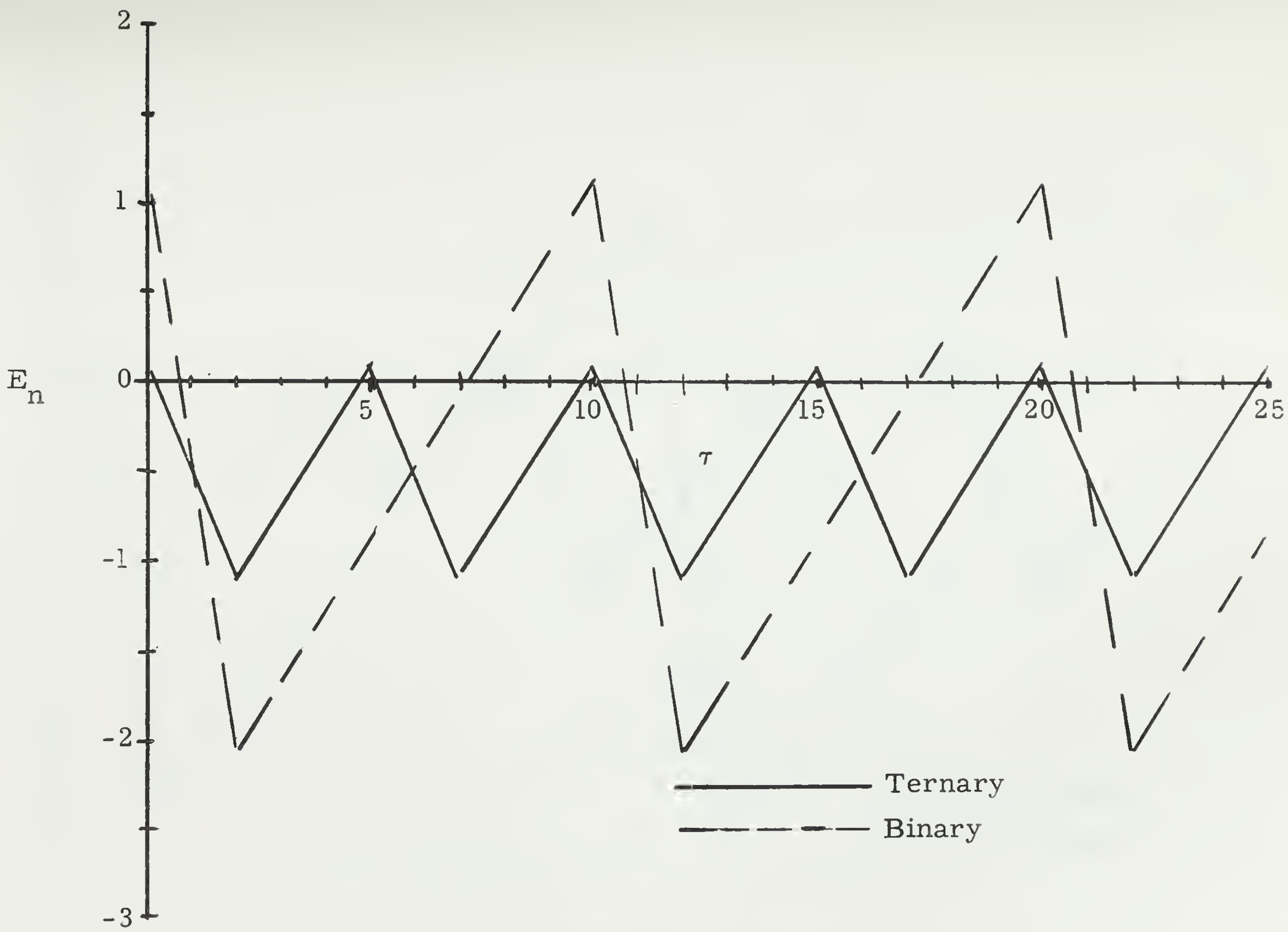


Figure 2-19. Null Error: $T_s = 1$, $r = 0.6$.

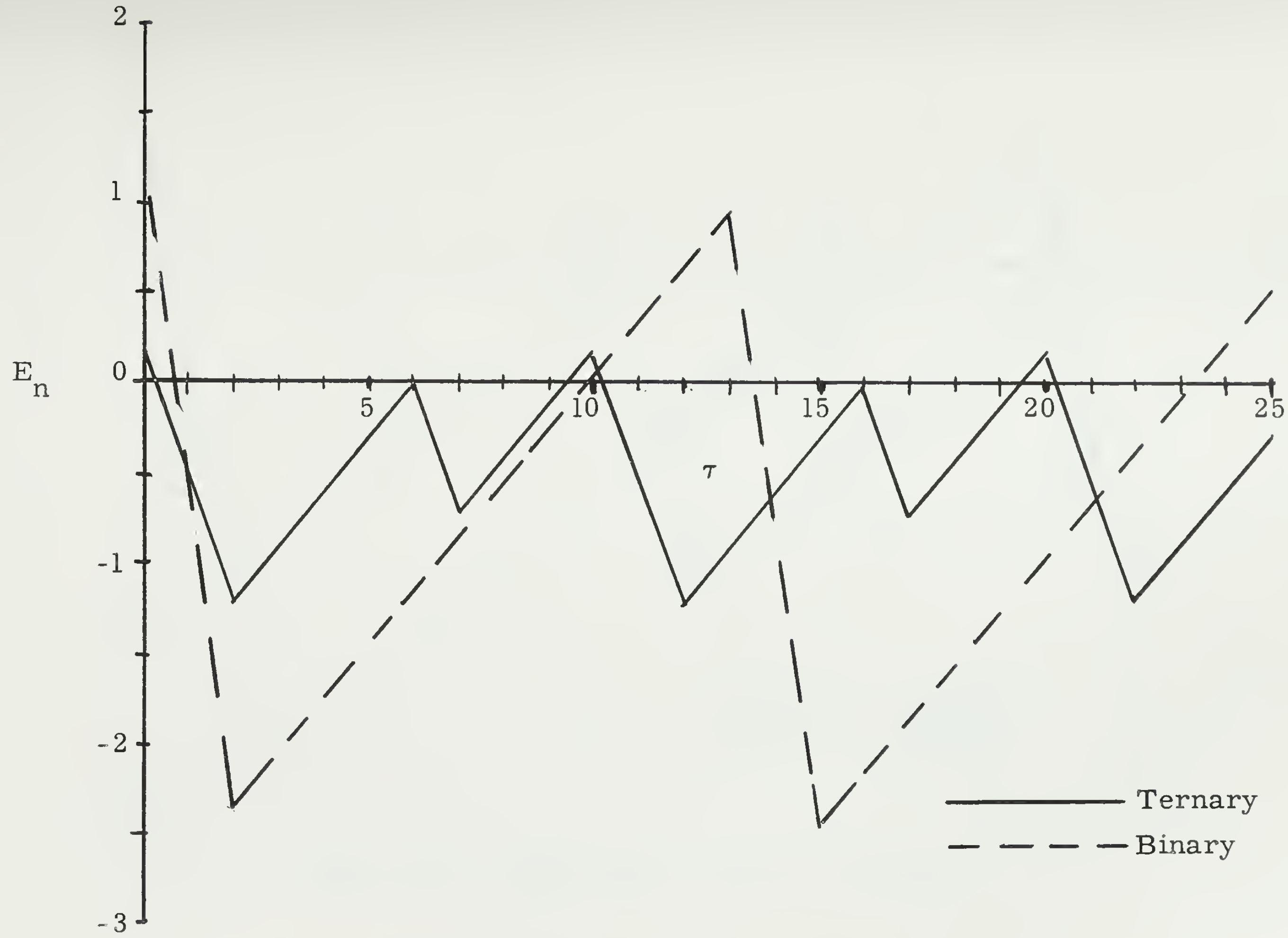


Figure 2-20. Null Error: $T_s = 1$, $r = 0.7$.

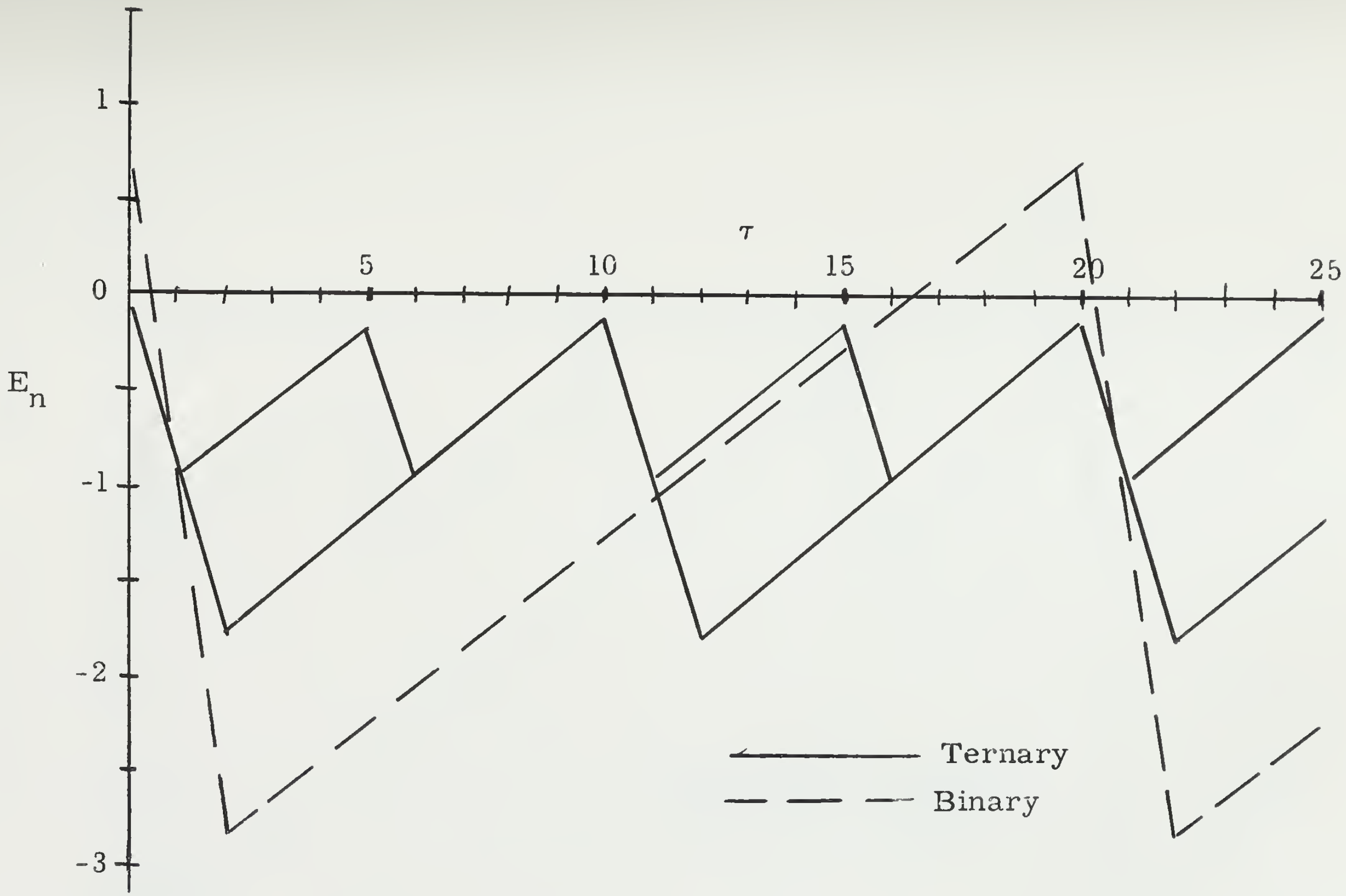


Figure 2-21. Null Error: $T_s = 1$, $r = 0.8$.

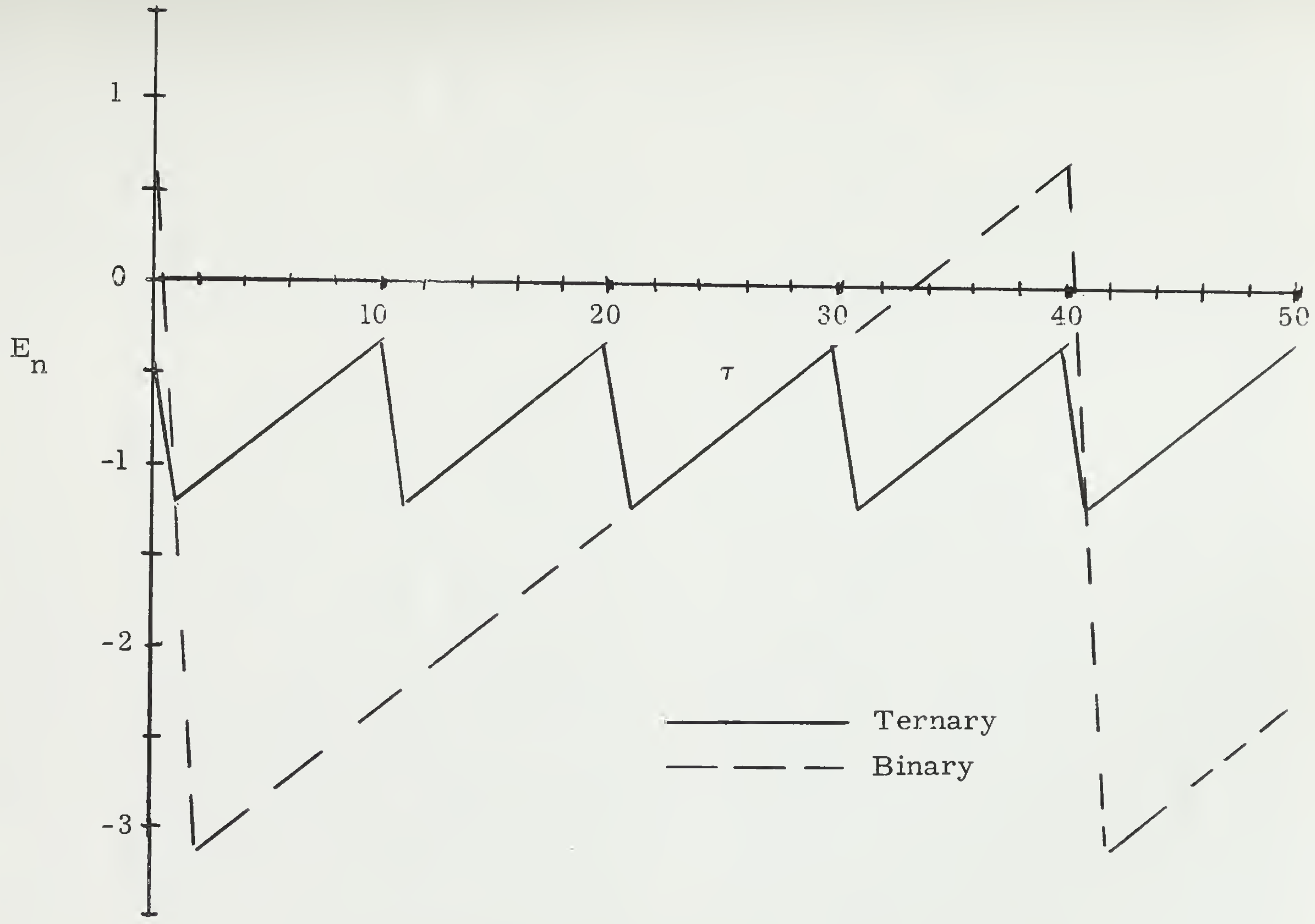


Figure 2-22. Null Error: $T_s = 1$, $r = 0.9$.

CHAPTER 3

MINIMIZING LIMIT CYCLE SIZE BY PROPER SELECTION OF SYSTEM PARAMETERS

3.1. GENERAL

In the previous chapter it was shown that a ternary pulsed pendulous accelerometer may operate in several modes of oscillation about a bias position. The bias, or average pendulum position, produces two distinct errors. First, the bias itself represents an error in velocity indication in that a certain amount of acceleration reaction is opposed by viscous friction in the process of the pendulum moving from its null position. This error is non-accumulative and its effect may even be negligible. Second, cross-coupling error is introduced by the bias angle. For small angle approximations, the cross-coupling error is the time integral of the product of the bias angle and the component of acceleration along the reference axis. Cross-coupling error is accumulative (i.e. it may grow with time). It may be minimized by maintaining the bias angle small, or may be "computed out" if the bias angle is known.

The additional error associated with the oscillation of the pendulum about its bias position, is oscillatory and therefore non-accumulative. The instantaneous error caused by this oscillation might even be negligible. However, these oscillations if large may cause the velocity information to be presented to the computer in an undesirable form. Additional errors may be generated by the computer which are accumulative. The nature and extent of these errors will not be investigated here. To minimize these errors it is desirable to maintain the pendulum in its lowest mode of oscillation. (A limit cycle is of the lowest mode is defined to be that limit cycle which encloses the least area of the non-dimensional phase plane. Thus for a non-dimensional acceleration of 2/7, a 1:2:1:3 limit cycle is of lower mode than a 2:5 or a 1:1:1:4 limit cycle.)

It is presupposed that certain characteristics are specified before initial design of a ternary pulsed accelerometer begins. Sample time,

maximum acceleration and physical size and weight limitations may be among the specifications. The designer is at liberty to select other parameters in proper combination to insure adequate performance. Such selection must be compatible with physical realizability at a "reasonable" cost.

It has been previously determined that dynamic behavior of a ternary system is determined by two non-dimensional quantities: sample time, T_S, and dead zone, ϕ_Δ. Knowledge of the effect of these two quantities on system performance, offers the designer a "crutch" in design analysis.

3.2. SELECTION OF NON-DIMENSIONAL SAMPLE TIME

Non-dimensional sample time is the ratio of sample time to the characteristic time of the viscous damped pendulum. It is desirable to select system parameters such that limit cycles of higher order are prevented. Since the value of non-dimensional sample time has a definite effect upon the modes of oscillation possible, it should be selected to prevent as many higher order limit cycles as possible.

Figures C-2 through C-6 show the freedom in position of a simple limit cycle about a switching line, plotted as a function of non-dimensional sample time. A simple limit cycle is defined to be a limit cycle that is complete after two switches between torque modes. Appendix C points out that if the freedom in position of a simple limit cycle is negative, then the simple limit cycle under consideration cannot exist. In general, as the non-dimensional sample time, T_S, is increased, more simple limit cycles are prevented. It is observed that the 2:2 limit cycle cannot be prevented by adjusting the non-dimensional sample time as the freedom in position of the limit cycle is positive for all values of sample time. Likewise the 1:N limit cycles, where N is any integer, cannot be prevented. The 1:N limit cycles are of the lowest mode and are desirable.

A value of five or greater for non-dimensional sample time will prevent all non-desirable simple limit cycles with the single exception of the 2:2 limit cycle. It is realized that the simple limit cycles represent only a few of the infinity of limit cycles possible. However, one would expect that the dynamic behavior of the pendulum would not change

greatly for a small change in acceleration. For this reason, it is felt that the simple limit cycles are representative of the over-all behavior of the pendulum.

3.3. SELECTION OF NON-DIMENSIONAL DEAD ZONE

The non-dimensional dead zone must be larger than the non-dimensional sample time in order to prevent torquing in a direction to aid acceleration. If it is larger than necessary, the bias angle will be larger than necessary and errors associated with the bias angle will be larger. It would appear that the best value for the non-dimensional dead zone is one which is larger than the non-dimensional sample time by an infinitesimal amount.

3.4. SELECTION OF DIMENSIONAL PARAMETERS

The physical dead zone that is inherent to the error detector is a limiting factor in maintaining the null error small. It would appear advisable that the dead zone not be made larger artificially, in order to implement a ternary pulsed system.

The key to the proper selection of dimensional parameters lies in the dimensional to non-dimensional transformations of Table 2-I. If the physical dead zone of the error detector is taken to be the value that will transform into a non-dimensional dead zone that is equal to the non-dimensional sample time, a system of equations is introduced which may be used in the selection of dimensional parameters. The fact that the non-dimensional dead zone must be larger than the non-dimensional sample time by a small amount, can be taken care of by final adjustment to the threshold detector.

Suppose that A_Δ is the dead zone expected from the error detector. Then by the transformations of Table 2-I:

$$\phi_\Delta = \frac{A_\Delta C^2}{MJ} \tag{3.1}$$

and

$$T_s = \frac{C\, t_s}{J} . \tag{3.2}$$

By desire T_s and ϕ_Δ are set equal

$$T_s = \phi_\Delta . \tag{3.3}$$

The applied torque, M, must be sufficient to overcome the reaction of the maximum expected acceleration. Therefore:

$$M = a_{max} \; P. \tag{3.4}$$

If a_{max}, A_Δ, and t_s are specified, six parameters remain to be fixed by the designer to satisfy the above four equations. The designer may select a value for T_s to give satisfactory dynamic performance, leaving five parameters to be adjusted to satisfy the four equations.

3.5. EXAMPLE

Suppose it is desired to design a ternary pulsed pendulous accelerometer with a maximum acceleration capability of 10^4 cm/sec^2 and with a sample time of 500 microseconds. The velocity quantum is the product of these two quantities or 5 cm/sec. A value of five is selected for non-dimensional sample time so that all undesirable simple limit cycles are prevented with the exception of the 2:2 limit cycle. Suppose also that the dead zone expected from the error detector is five microradians. Substituting these values into Equations (3.1) through (3.4):

$$\frac{C}{M} = 100 \text{ sec.} \tag{3.5}$$

$$\frac{C}{J} = 10^4 \text{/sec} \tag{3.6}$$

$$\frac{M}{P} = 10^4 \text{ cm/sec}^2 . \tag{3.7}$$

If P is arbitrarily selected to be one gm-cm., Equations (3.5) through (3.7) can then be solved for the remaining unknowns:

$$M = 10^4 \text{ dyne-cm}$$

$$J = 100 \text{ gm-cm}^2$$

and

$$C = 10^6 \text{ dyne-cm-sec.}$$

Figures 3-1 through 3-11 show the steady state phase plane response of such a system for various values of input acceleration. Figures 3-12 through 3-15 show the null error versus time response for the phase plane responses of Figures 3-1 through 3-11.

The phase plane plots show that the pendulum is in its lowest mode of oscillation except when the acceleration is in the vicinity of one half the maximum. This could be expected in that the 2:2 limit cycle has not been eliminated. If the 2:2 limit cycle does not produce excessive error in the computer, or if the accelerometer is not expected to operate in the vicinity of one half the maximum, then the system is satisfactory in that sufficient higher order limit cycles have been eliminated.

3.6. LIMITATIONS TO THE METHOD

The authors admit that the previous example has been somewhat "rigged" so that realistic answers could be obtained for dimensional parameters. The specifications could be such that Equations (3.1) through (3.4) produce dimensional quantities that are not physically realizable if a value of five is selected for T_s. One may have to settle for a value much less than five for T_s, in which case another method will have to be employed to maintain the limit cycles in their lowest mode. Such methods are the subject of the next chapter.

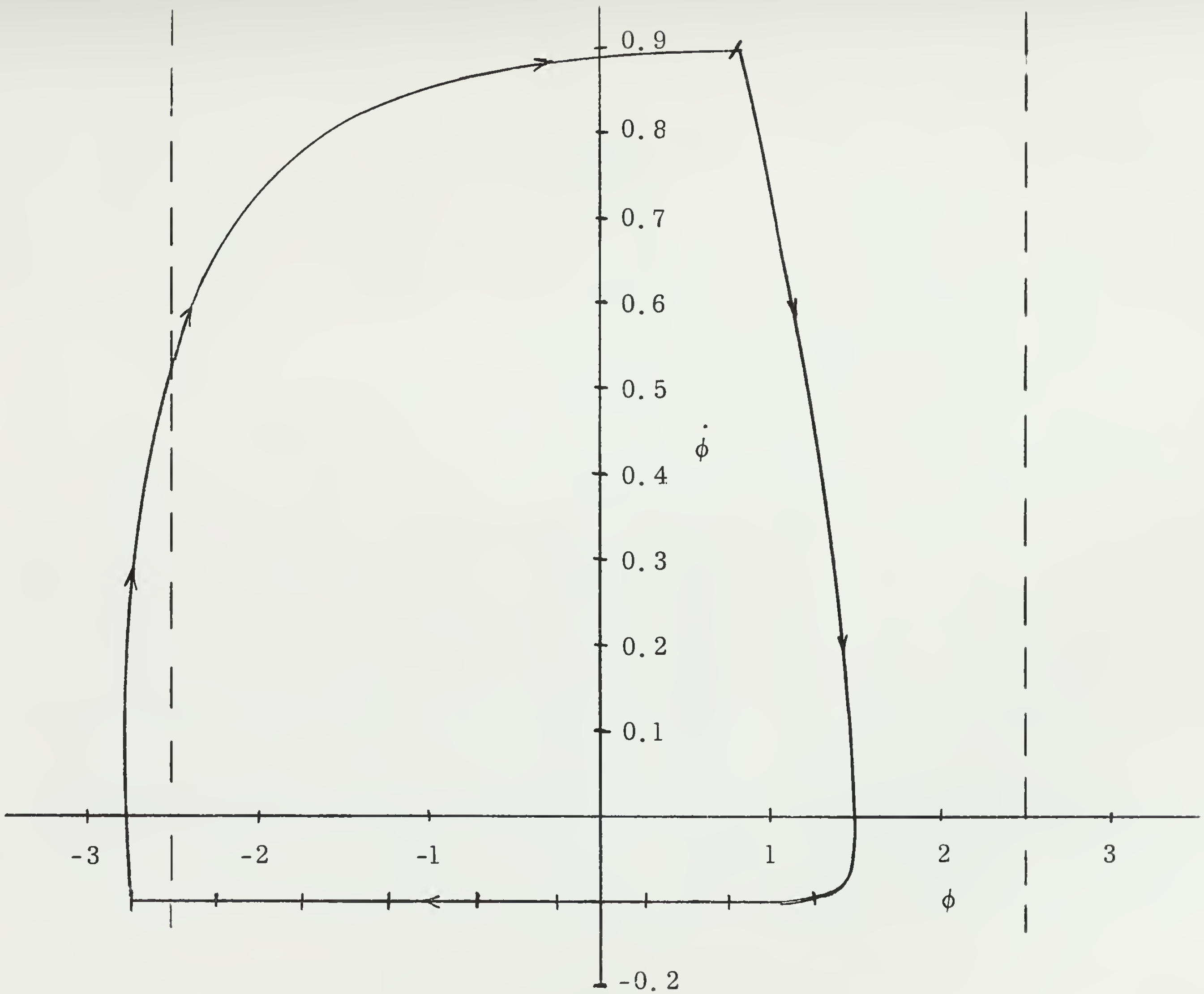


Figure 3-1. Steady State Limit Cycle: $T_s = 5$, $r = 0.1$.

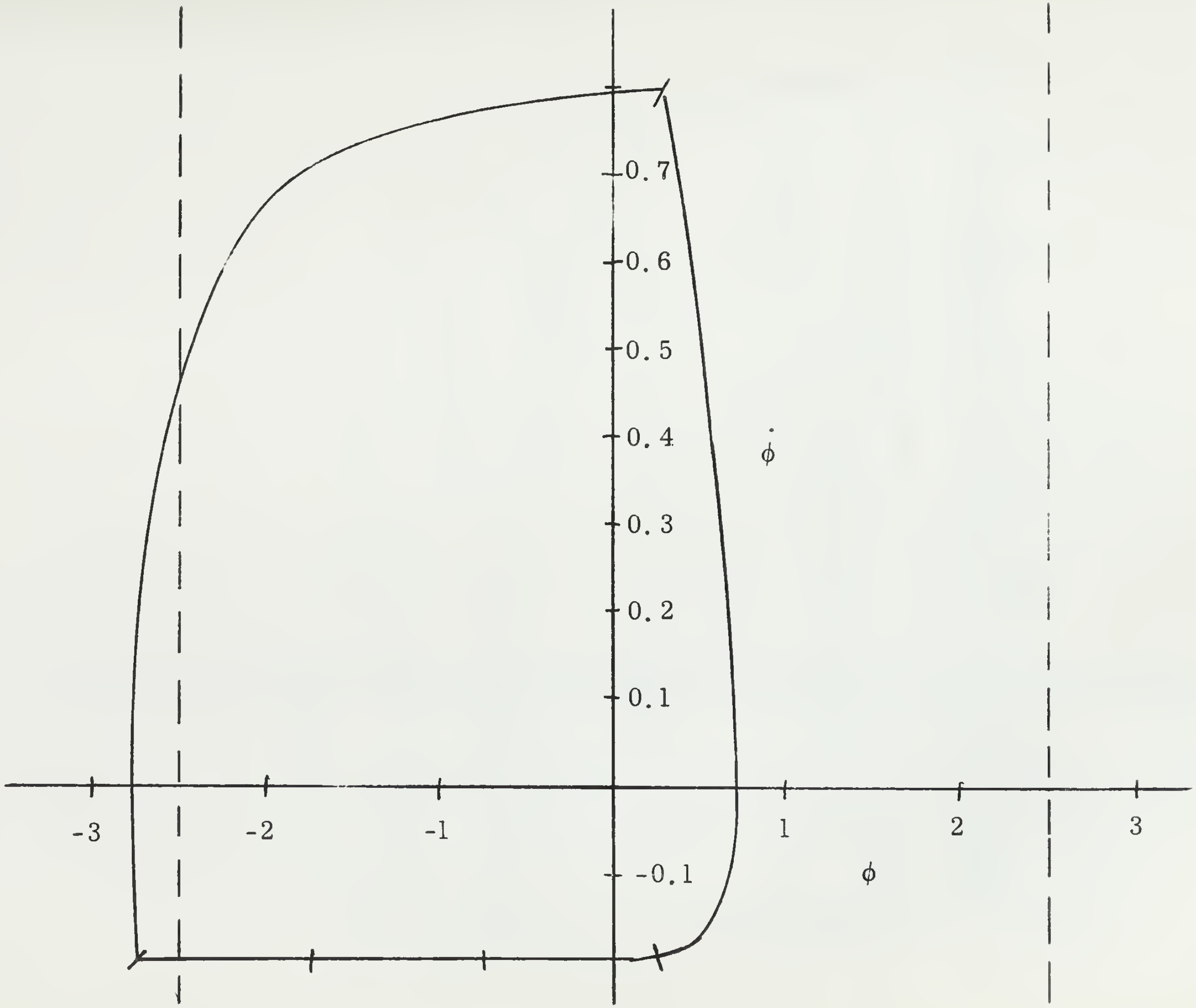


Figure 3-2. Steady State Limit Cycle: $T_s = 5$, $r = 0.2$.

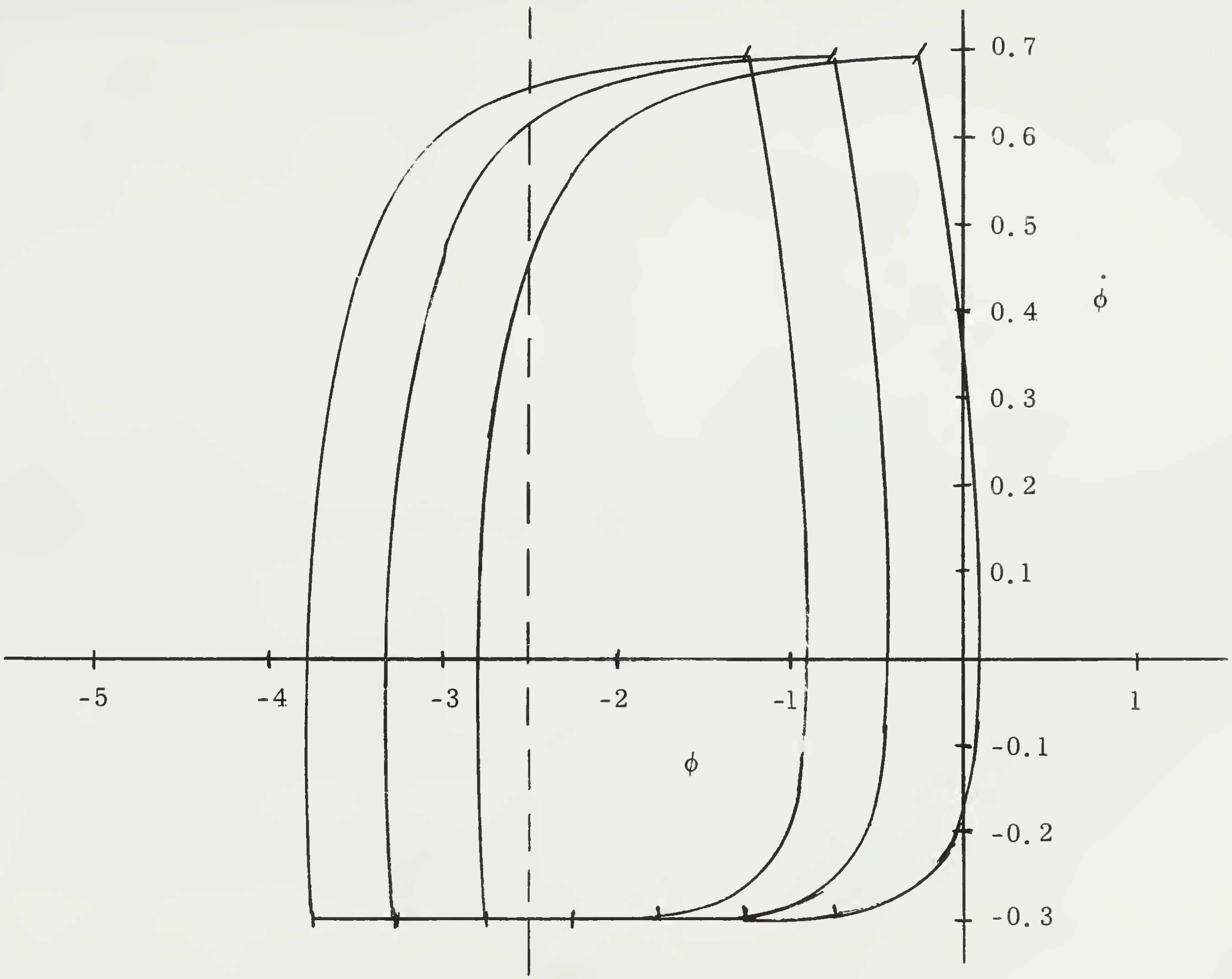


Figure 3-3. Steady State Limit Cycle: $T_s = 5$, $r = 0.3$.

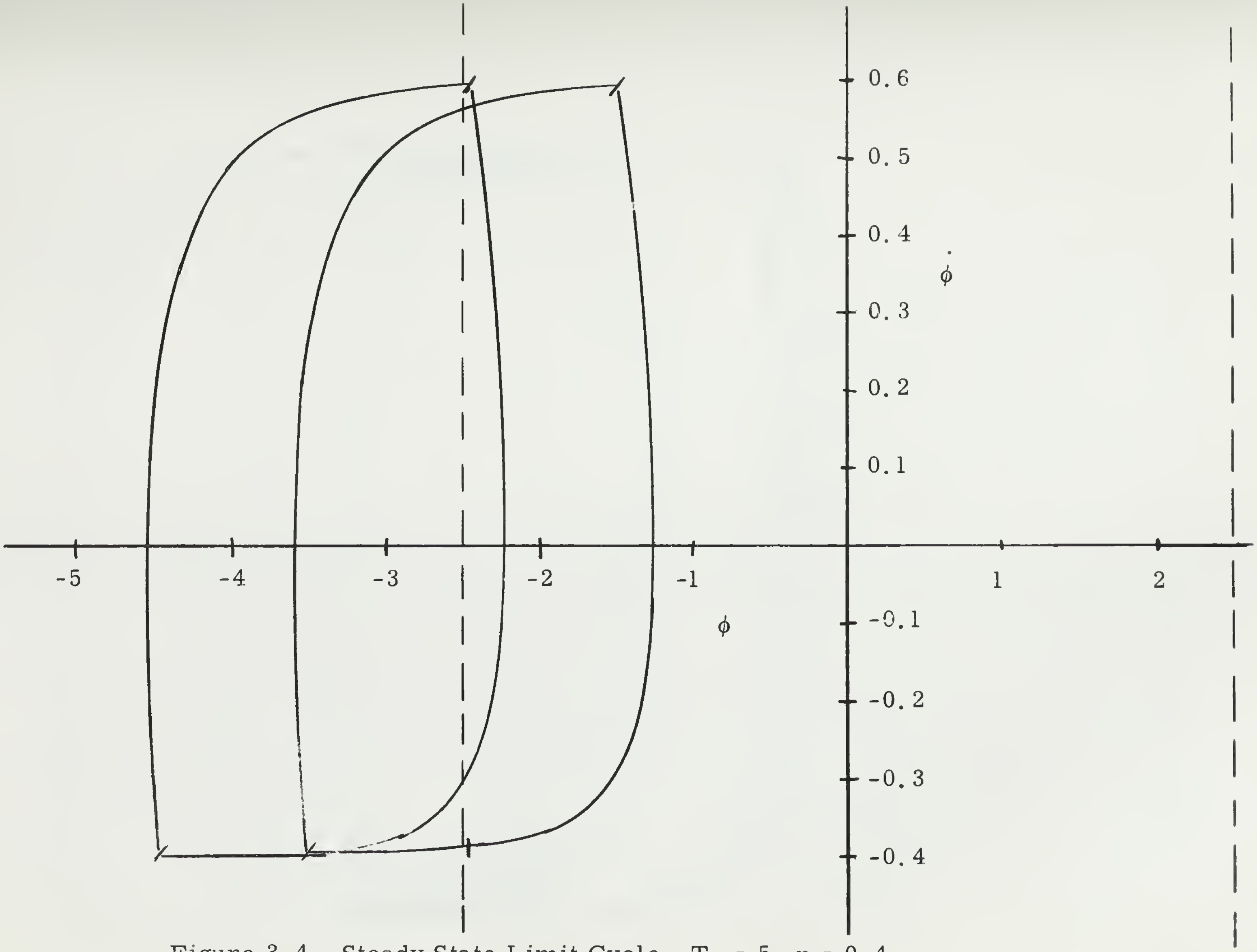


Figure 3-4. Steady State Limit Cycle: $T_s = 5$, $r = 0.4$.

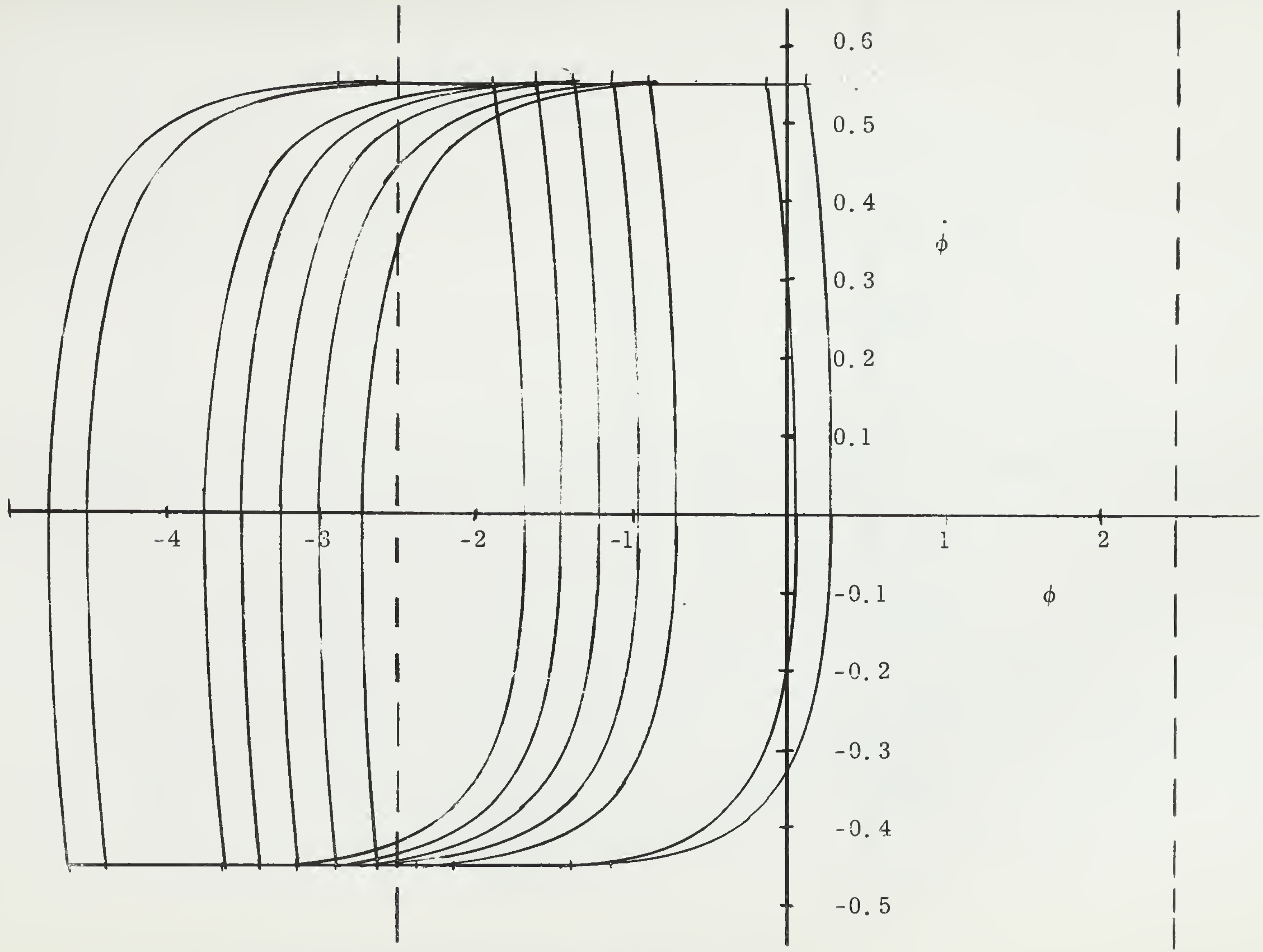


Figure 3-5. Steady State Limit Cycle: $T_s = 5$, $r = 0.45$.

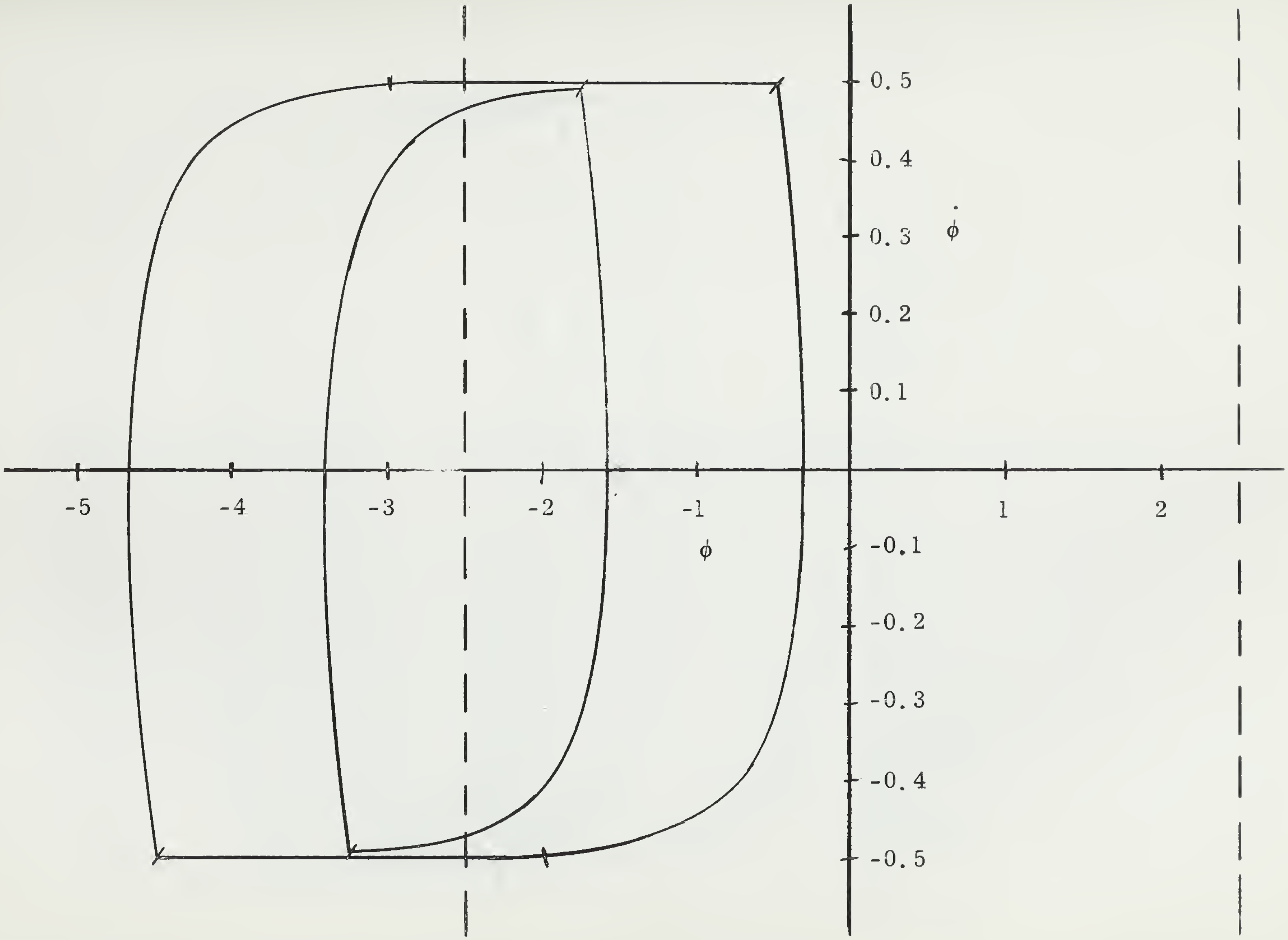


Figure 3-6. Steady State Limit Cycle: $T_s = 5$, $r = 0.5$.

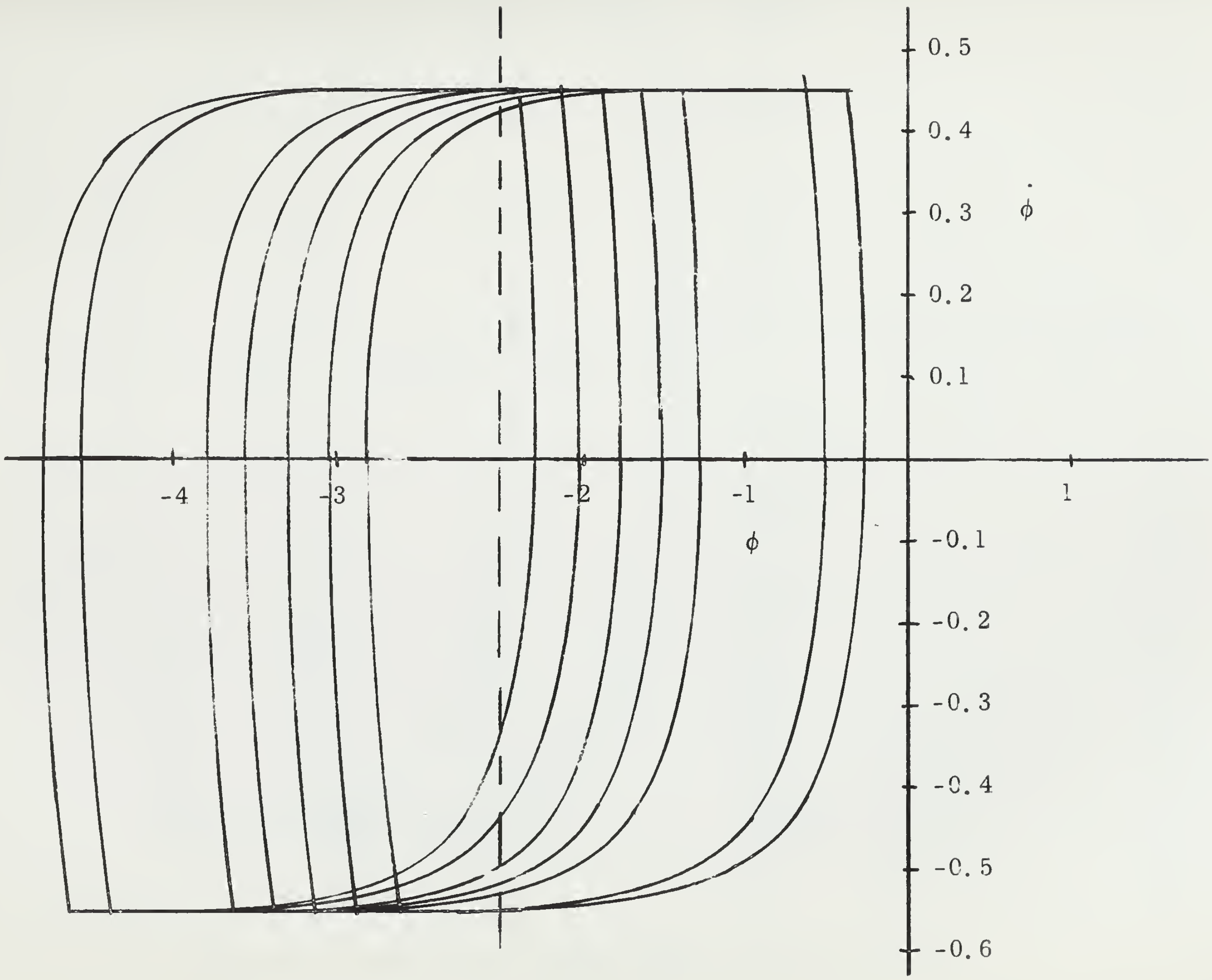


Figure 3-7. Steady State Limit Cycle: $T_s = 5$, $r = 0.55$.

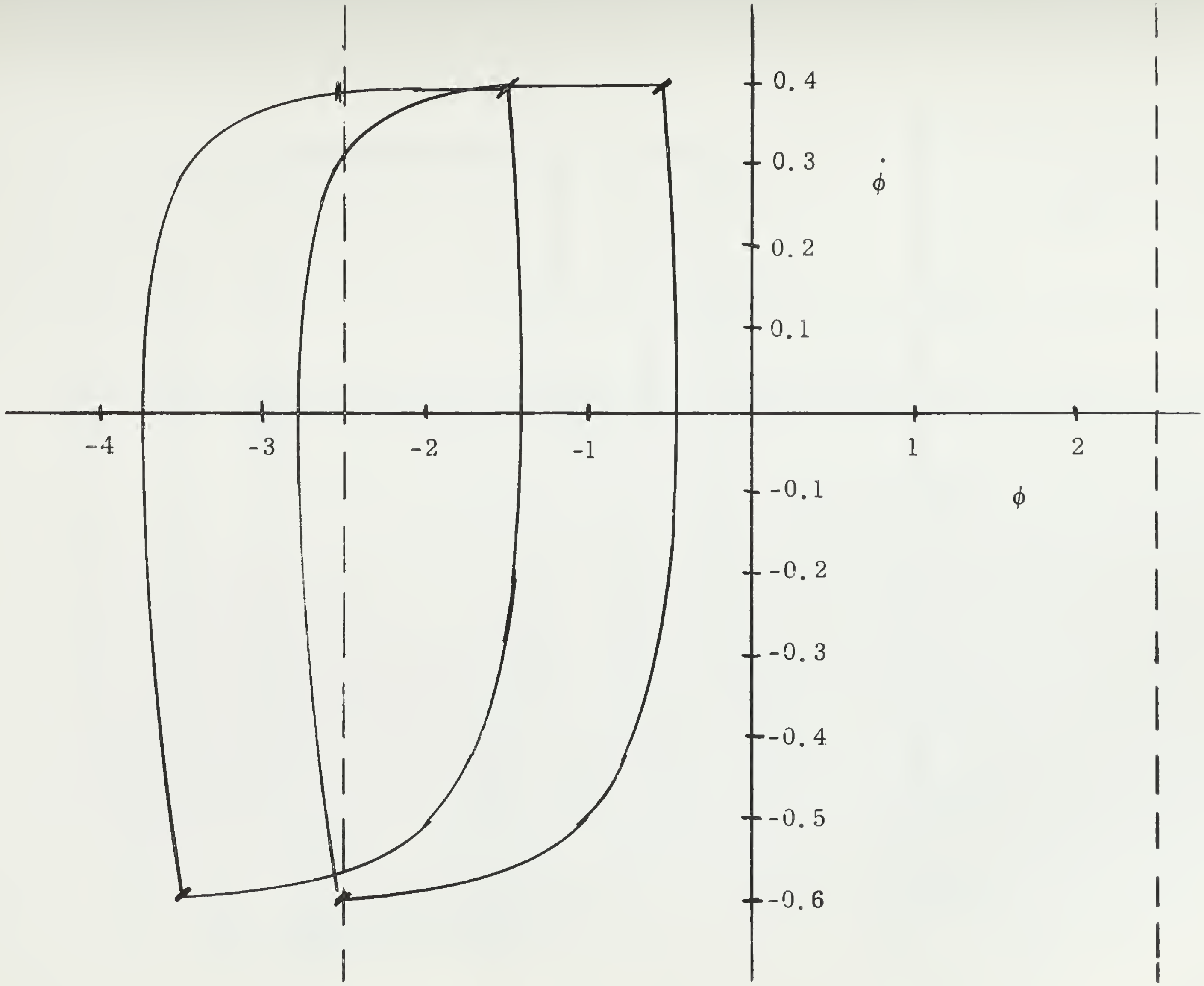


Figure 3-8. Steady State Limit Cycle: $T_s = 5$, $r = 0.6$.

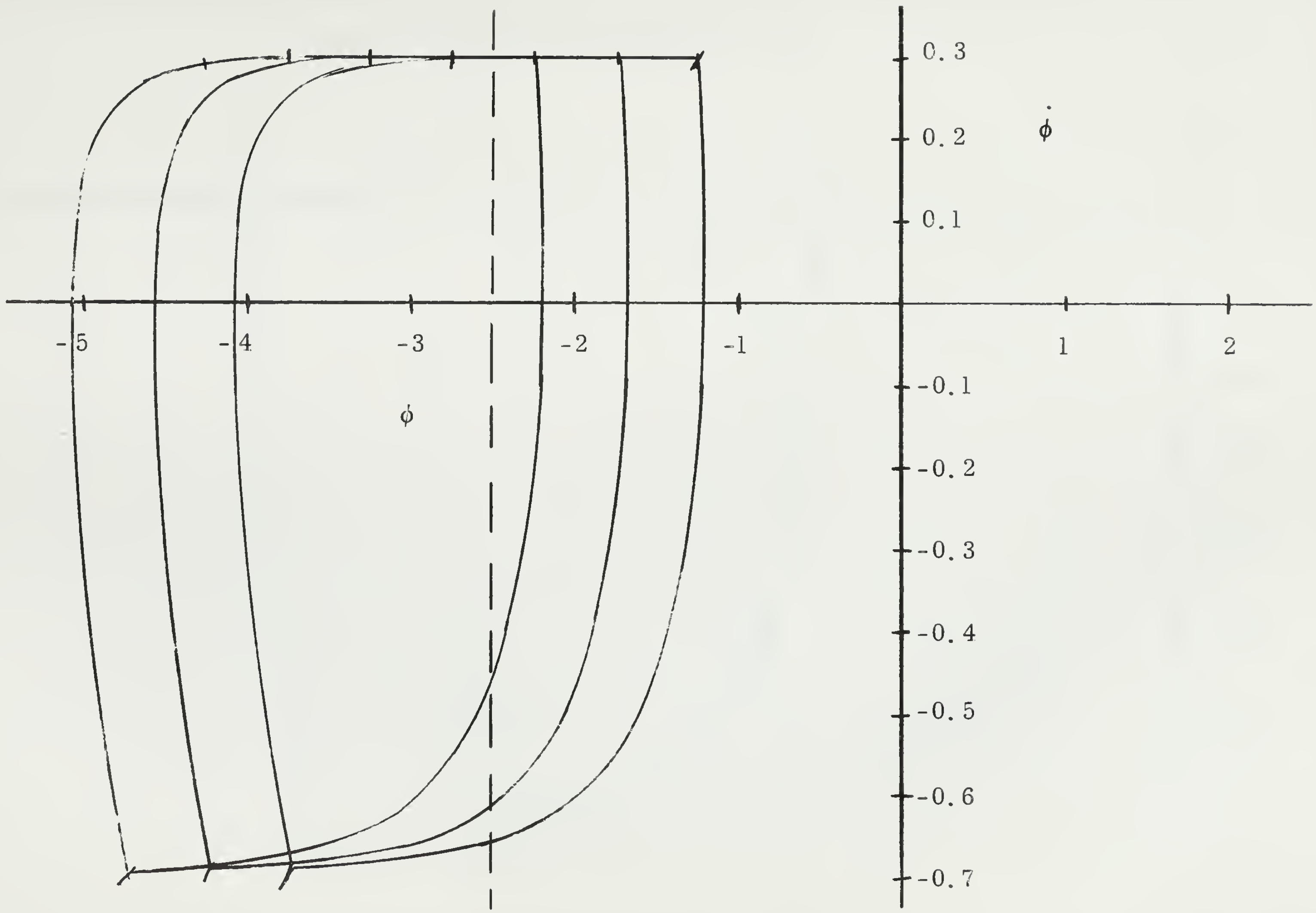


Figure 3-9. Steady State Limit Cycle: $T_s = 5$, $r = 0.7$.

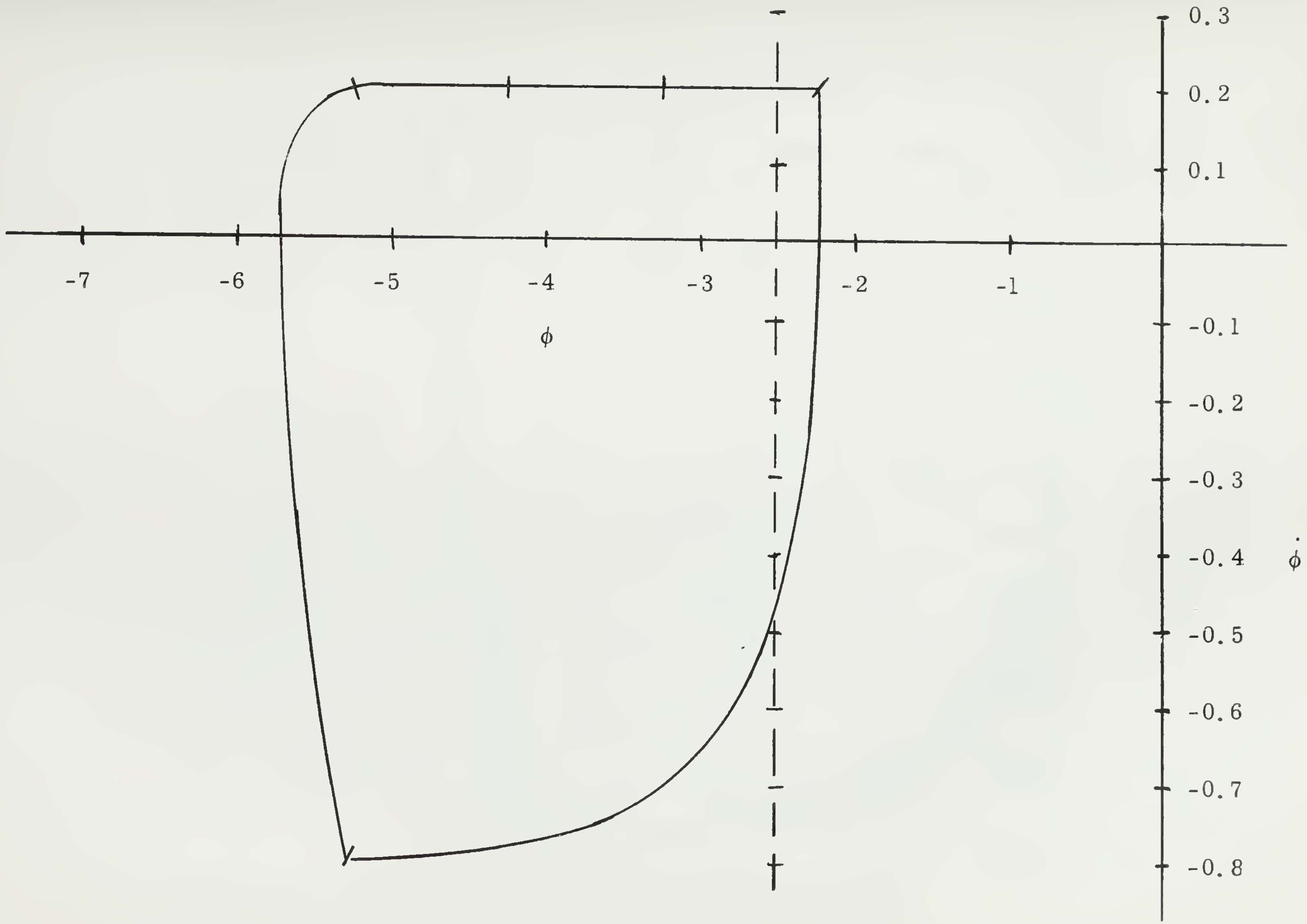


Figure 3-10. Steady State Limit Cycle: $T_s = 5$, $r = 0.8$.

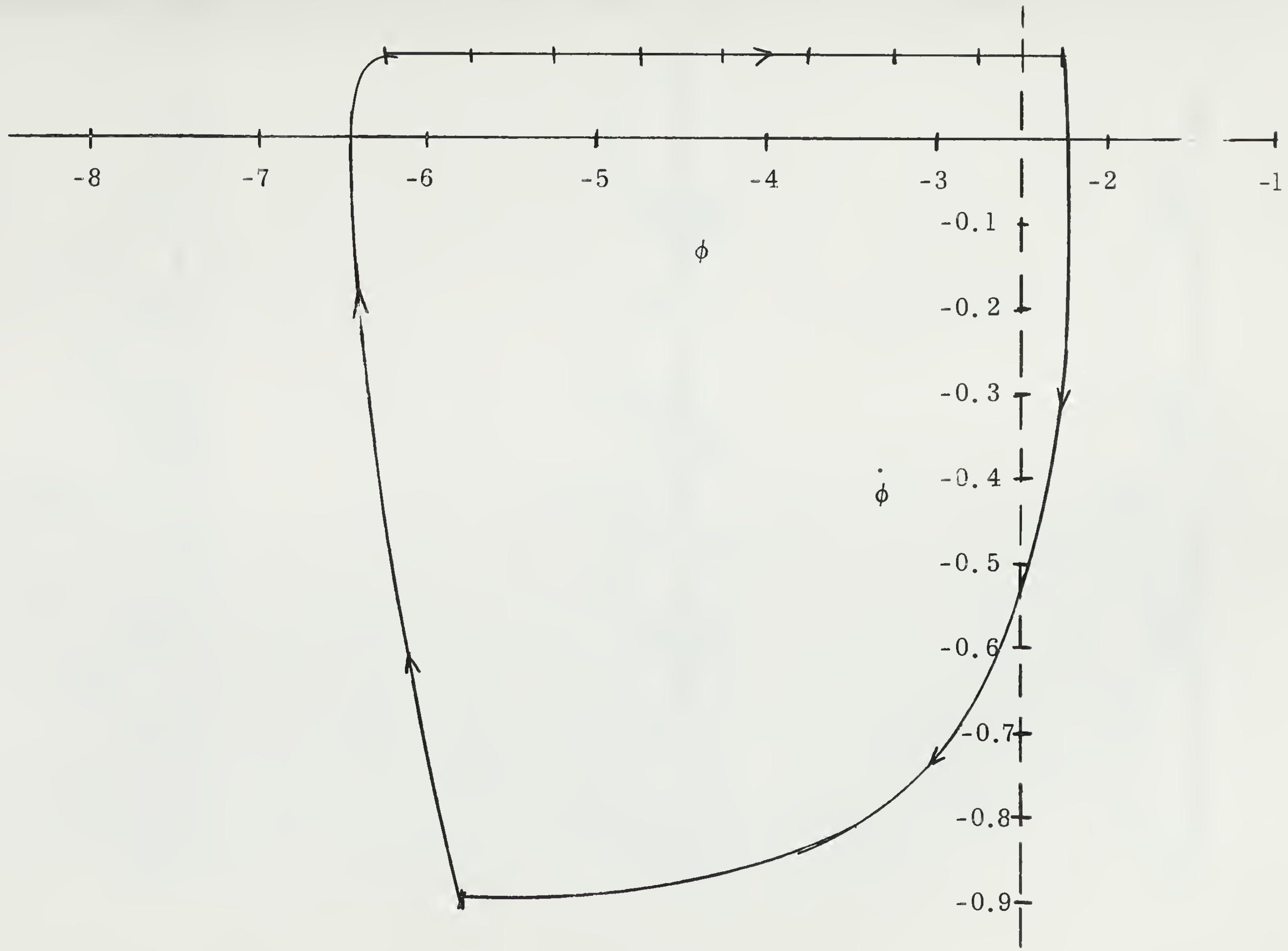


Figure 3-11. Steady State Limit Cycle: $T_s = 5$, $r = 0.9$.

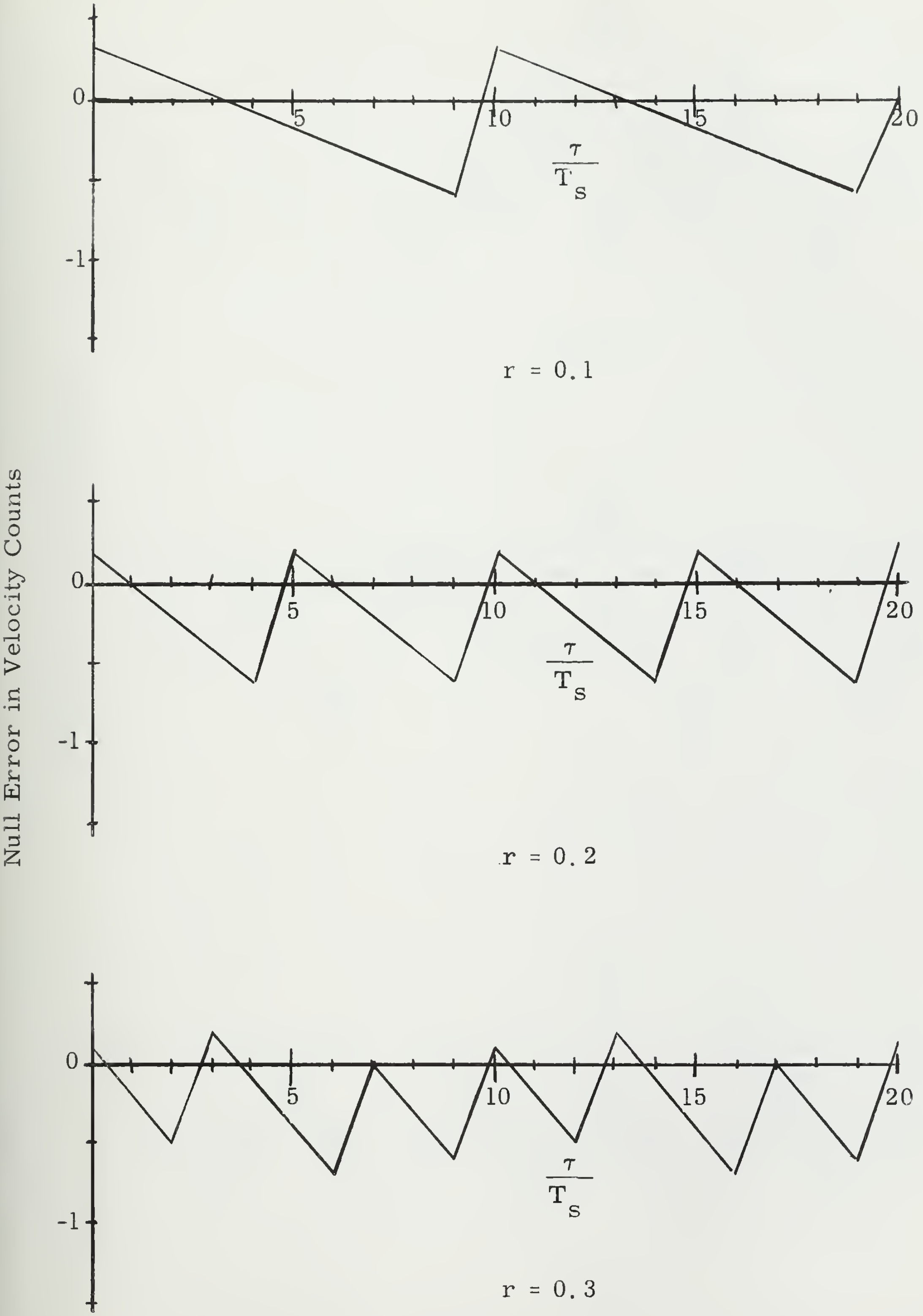


Figure 3-12. Null Error: $T_s = 5$, $r = 0.1, 0.2, 0.3$.

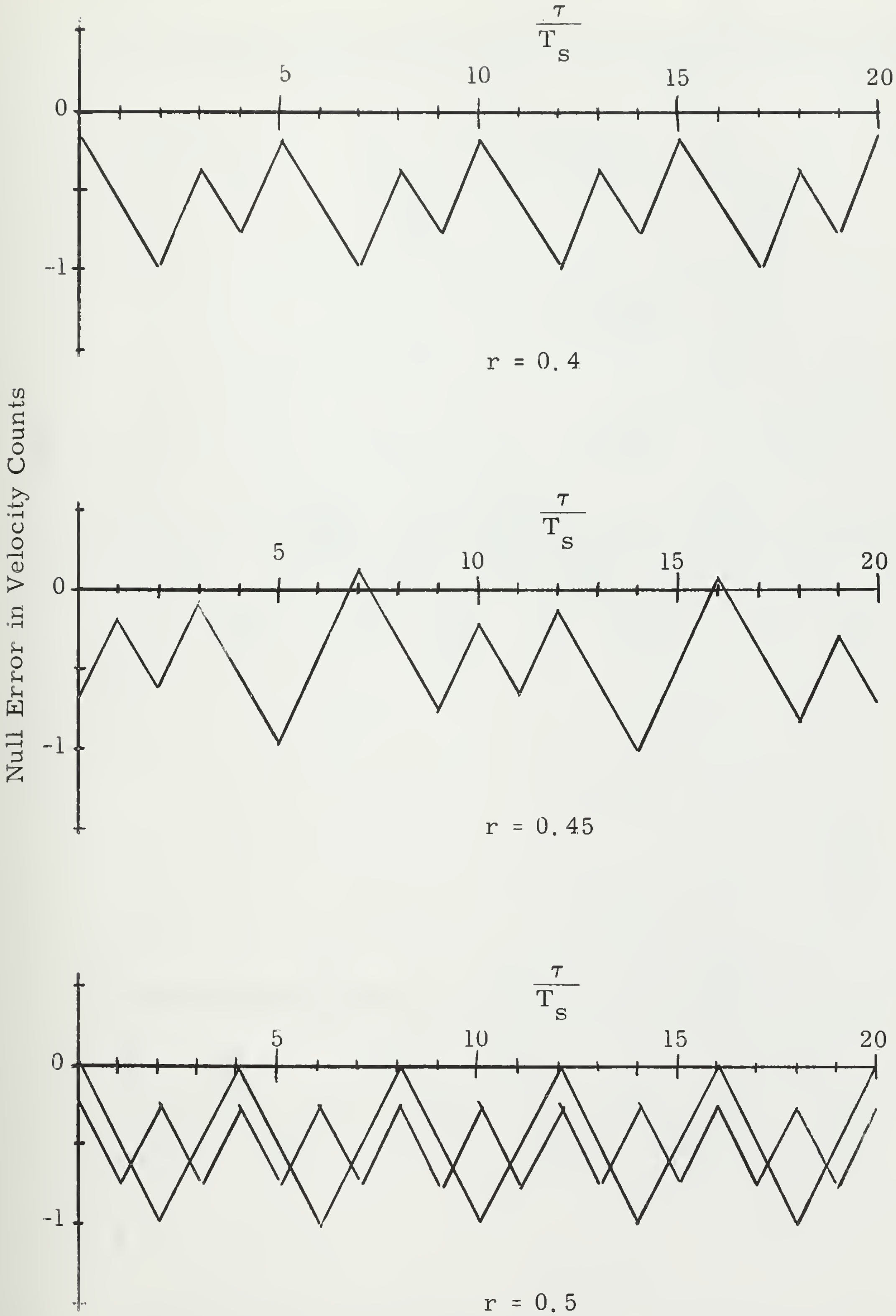


Figure 3-13. Null Error: $T_s = 5$, $r = 0.4$, 0.45, 0.5.

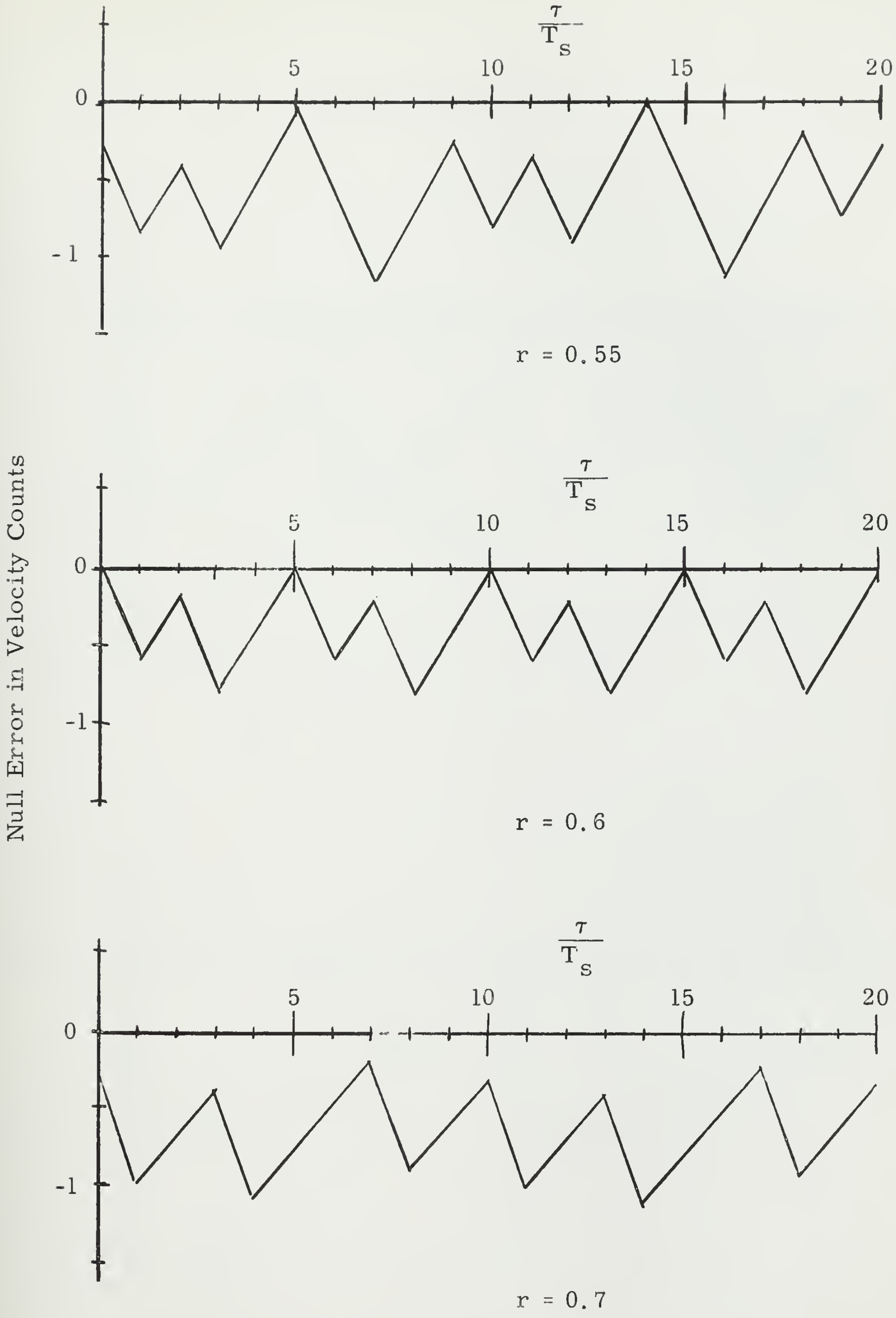


Figure 3-14. Null Error: $T_s = 5$, $r = 0.55, 0.6, 0.7$.

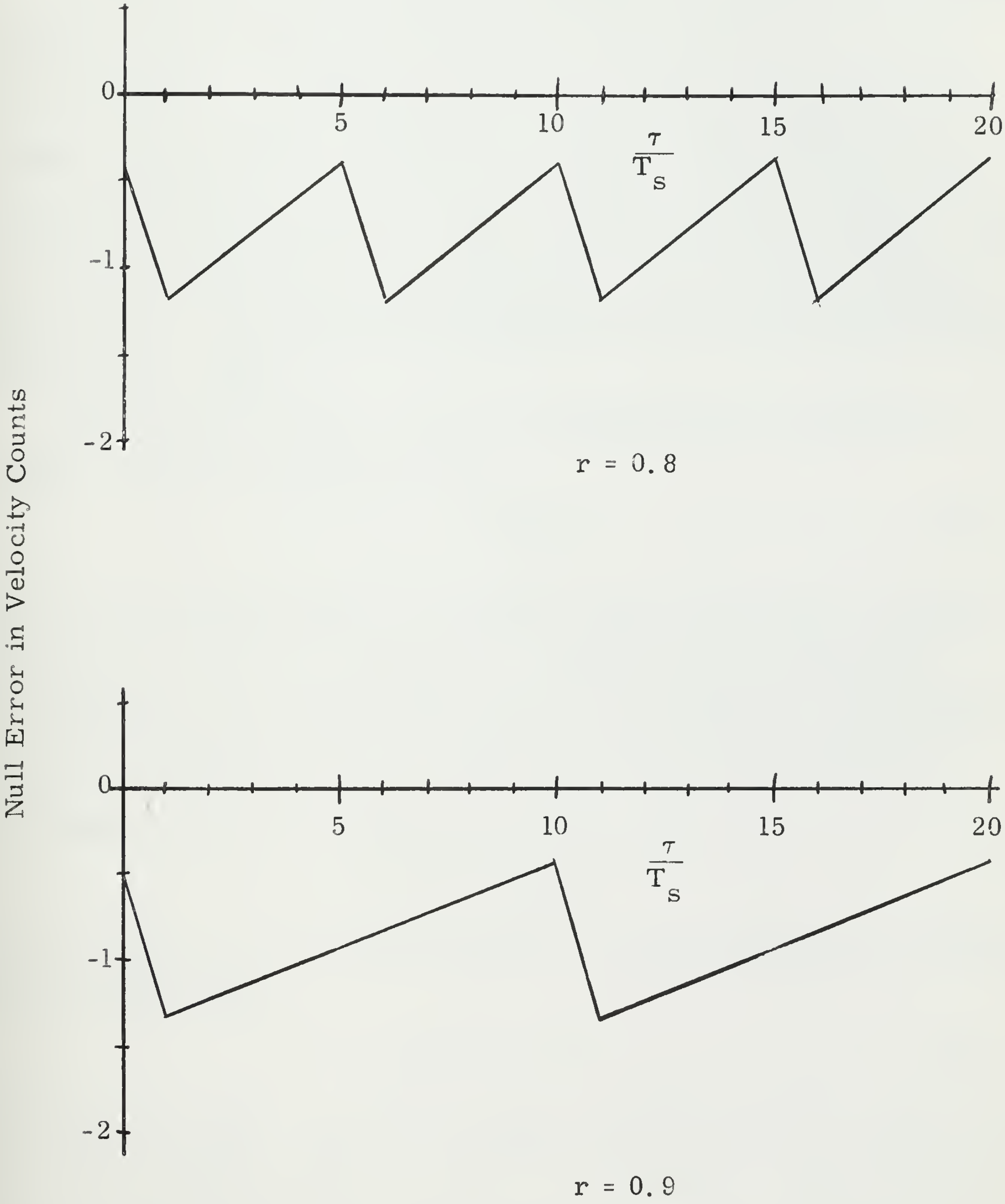


Figure 3-15. Null Error: $T_s = 5$, $r = 0.8, 0.9$.

CHAPTER 4

COMPENSATION OF TERNARY MODE PULSE TORQUED PENDULOUS INTEGRATING ACCELEROMETERS

4.1. INTRODUCTION

Torquing only in the direction to oppose the acceleration reaction torque and torque pulsing in a routine of the lowest order are the envisioned benefits of ternary mode pulse torquing. By routine of the lowest order is meant torquing with a routine which keeps the area enclosed by the limit cycle as small as possible. When the input acceleration-maximum acceleration ratio r, is less than one-half, this will be done if the restoring torque pulse is the shortest time possible, which is T_s, the non-dimensional sampling period. For values of r greater than one-half, the non-torquing period should have the shortest possible time duration, again T_s.

Any rational value of r can be represented by a fraction with integer numerator and denominator.

$$r = \frac{K}{K + L}\,, \tag{4.1}$$

where K is the total number of torquing sample periods per counting cycle, and L is the total number non-torquing sample periods per counting cycle. A counting cycle is of K + L sample periods duration and is the smallest number of sample periods which accurately represents r. Both K and L can be the number of sampling periods in more than one torque or non-torque periods. Or

$$K = K_1 + K_2 + K_3 + \cdots\cdots\cdots \tag{4.2}$$

where K_1, K_2, K_3, $\cdots\cdot$ are the number of sampling periods in individual torque periods, and

$$L = L_1 + L_2 + L_3 + \cdots\cdots\cdots \tag{4.3}$$

where L_1, L_2, L_3, $\cdots\cdot$ are the number of sampling periods in individual non-torque periods. When r is less than one-half the lowest order counting cycle can be produced if K_1, K_2, K_3, $\cdots\cdot$ equal one. Likewise, when r is greater than one-half, the lowest order counting cycle can be produced if

L_1, L_2, L_3, $\cdots$ equal one.

The above criteria will be the goal in compensating pulse torqued pendulous accelerometers. In achieving this goal errors other than the error due to higher order counting cycles will be introduced. These new errors should be smaller than the errors produced by the larger limit or counting cycles, or they must be of a nature such that the total velocity stored in the computer can be corrected for the errors introduced by the compensation.

The larger order limit cycles found in sample data bang-bang servos are the result of lag introduced in the system by discrete sampling rather than continuous sampling. Thus, lead must be introduced into the error circuit. In nonlinear servo theory, on the error-error rate phase-plane, a servo with lead in the error circuit and a dead zone has its switching lines represented in the manner shown in Figure 4-1. In the case of sample data bang-bang servos, switching from one torque state to another occurs at the first sampling instant after crossing the switch line.

In servo mechanisms in which the error magnitude and sense is represented in an analog manner by a voltage, lead compensation can be inserted by active or passive electronic networks. In pulse torqued integrating accelerometers null error is analoged by the voltage output of the microsyn. Representative sampling frequencies are one to five kilocycles (1000 to 5000 c.p.s.). Maximum frequencies at which microsyns have been found to be excited are also one to five kilocycles. Phase distortion increases as the exciting frequency is increased. The error analog is of poor quality. In existing binary pulse torqued pendulous accelerometers, the microsyn null position error detector is used only to determine the sense of the error. Hence, because of poor error analog quality, compensation of the error analog signal is not practical.

What is available in the system is the error sense sampled at equal periods. This suggests compensation using logic techniques. To simulate the lead switch lines shown in Figure 4-1, error sense, magnitude, and rate must be obtainable. The best magnitude resolution which can be expected from a microsyn type position error detector is about the width of the microsyn dead zone. Error rate is not readily available. The most which can be obtained readily from microsyn null error detectors at the desired sampling frequency is pendulum position

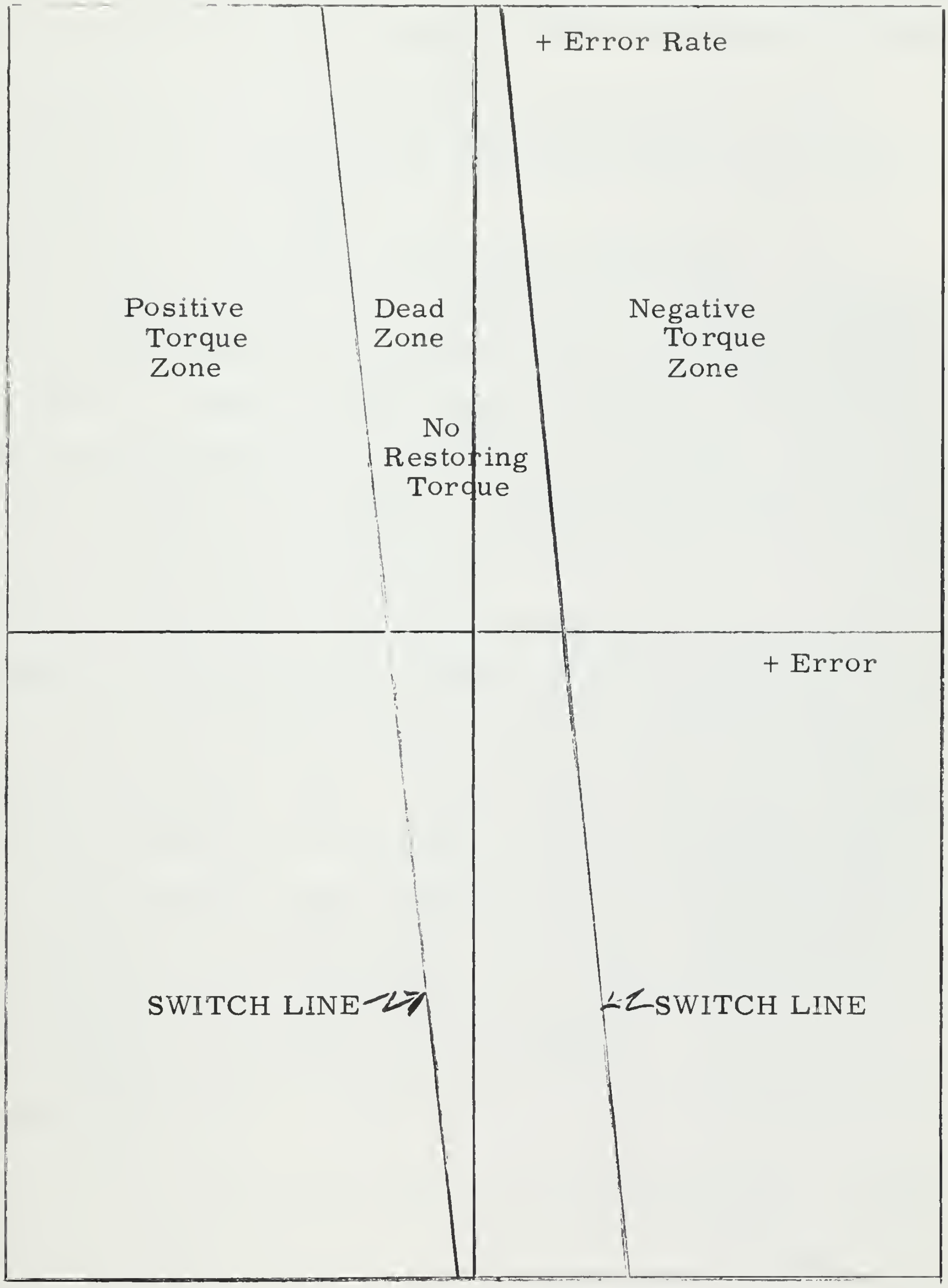


Figure 4-1. Phase Plane Representation of a Second Order Position Servo With Dead Zone and Lead.

in zones. These zones are the dead zone and magnitude zones of detectable sense.

The above zones can be determined and controlled in width with limited ability by passing the microsyn output signal through a phase detector to determine sense. Then the signal can be applied to biased level detectors to set zone magnitude.

Figure 4-2 shows the phase plane divided into five zones. Following are logic techniques in using these zones to insert lead compensation into the system.

4.2. ALTERNATING PULSE ZONE COMPENSATION

Lead compensation can be interpreted as a means of anticipating required actions and carrying them out sooner than a non-compensated system would. Figure 4-1 shows that the magnitude of the lead effect should be proportional to the magnitude of the velocity. This is not possible with available error detectors at the information rate required. Where our system does not allow a pure lead effect, the lead producing zone can produce a quasi lead effect.

A zone of sampling periods of alternating torquing and not torquing periods will anticipate both the change from the non-torquing dead zone to the continuous torquing zone and vice versa. Considering positive input accelerations, it is obvious for values of input acceleration - maximum acceleration ratio, r, greater than 0.5 that more torquing periods are required in a counting cycle than non-torquing periods. Hence, ϕ will increase until switching is about the boundary between the continuous and alternating zones, the $-\phi_L$ line. Similarly for values of r less than 0.5 the switching will be about the boundary between the alternating zone and the dead zone, the $-\phi_D$ line.

Previously the non-dimensional equations of motion of the pendulum were determined to be

$$\phi = (\delta - r)(\tau + e^{-\tau} - 1) + (1 - e^{-\tau})\dot{\phi}_o + \phi_o \qquad (4.4)$$

$$\dot{\phi} = (\delta - r)(1 - e^{-\tau}) + \dot{\phi}_o e^{-\tau} \qquad (4.5)$$

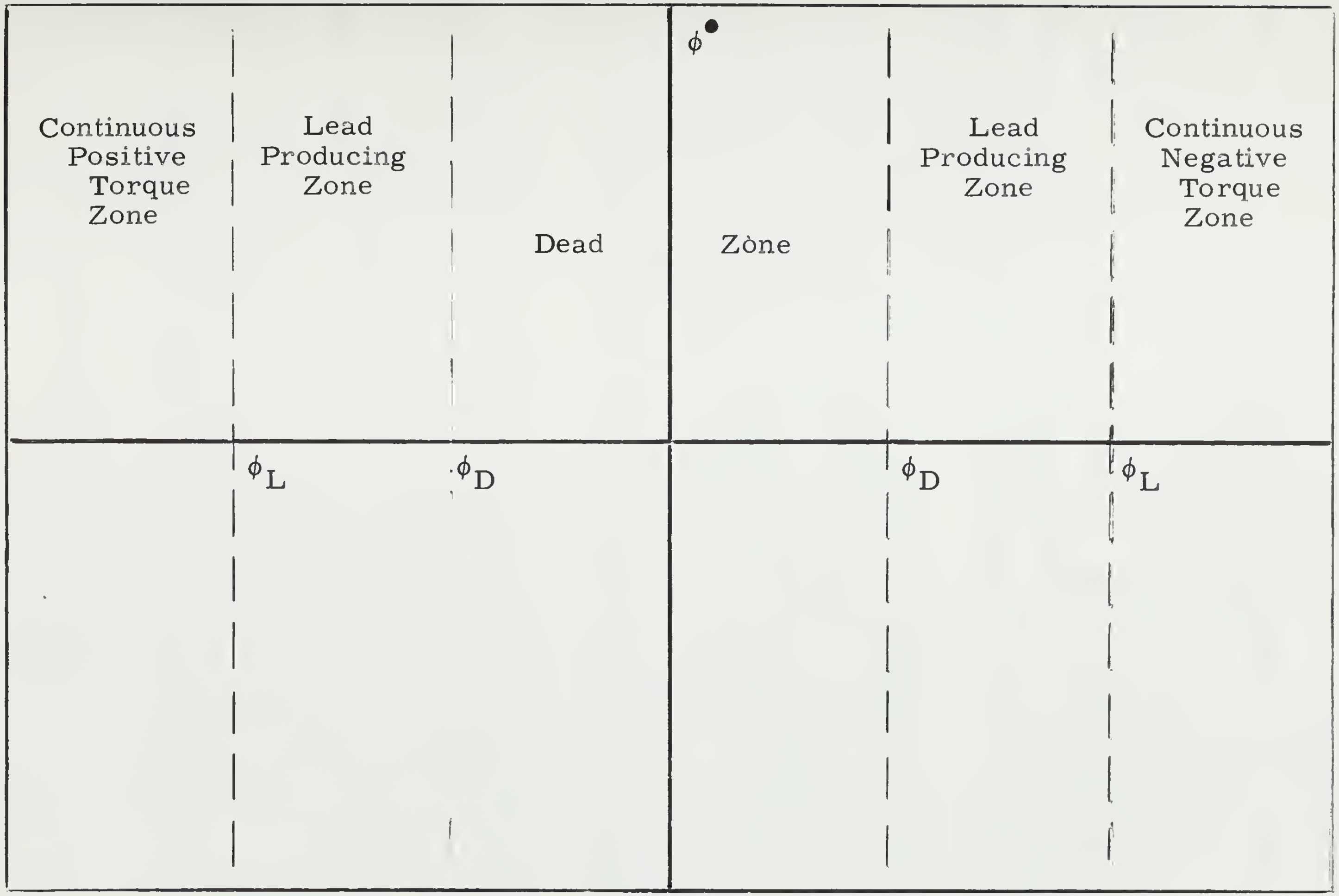


Figure 4-2. Division of the Phase Plane Into Zones Showing the Lead Producing Zones.

Except for the situation in which a very quick change in the acceleration has reversed the sense of the acceleration such that the acceleration reaction torque and the applied error reduction torque are in the same direction, the magnitude of maximum $\dot{\phi}$ in the alternating torquing zone is seen from equation (4.2) to be $(\delta - r)$. One-half is the maximum value of r when switching from the ϕ_D line. Since when moving from the dead zone into the alternating torque zone δ equals zero, the maximum $\dot{\phi}$ associated with switching about the $-\phi_D$ line in the alternating torque zone is 0.5. Similarly, since 0.5 is the minimum value of r and $\delta = 1$ when switching about the $-\phi_L$ line and moving from the continuous torque zone into the alternating torque zone, the maximum magnitude of $\dot{\phi}$ associated with switching about the $-\phi_L$ line in the alternating torque zone is 0.5.

If $\dot{\phi}$ is greater than 0.5 in the alternating torque zone the switching locus is moving from one boundary to the other because of a change in r from larger than 0.5 to smaller than 0.5, or vice versa. In this case switching in the alternating zone is not necessary. If $\dot{\phi}$ is less than 0.5 switching is mandatory. This will be assured if the width of the alternating zone is made at least equal to the maximum $\dot{\phi}$ desired to be "caught" in the alternating zone times the non-dimensional sampling period T_s. The minimum value of $\phi_L - \phi_D$ acceptable is $0.5\ T_s$.

The width of the dead zone must be adjusted so that, for very small values of r, the pendulum does not pass into the alternating zone where the magnitude of $(\delta - r)$ is greater than 1. Refer to the conditions shown in Figure 4-3 for the limiting case of small r, that is, r approximately equal to zero. ϕ_o equals $-\phi_D$, $\dot{\phi}_o$ approximately equals zero. ϕ_2 is less than $+\phi_D$. ϕ_2 is the point of furthest penetration into the positive ϕ plane before the pendulum is driven back to the negative ϕ plane. By equations (4.4) and (4.5), with a torque pulse of magnitude $(\delta - r)$ approximately equal 1, the position one sampling period later is

$$\phi_1 = T_s + e^{-T_s} - 1 - \phi_D \tag{4.6}$$

$$\dot{\phi}_1 = 1 - e^{-T_s} \tag{4.7}$$

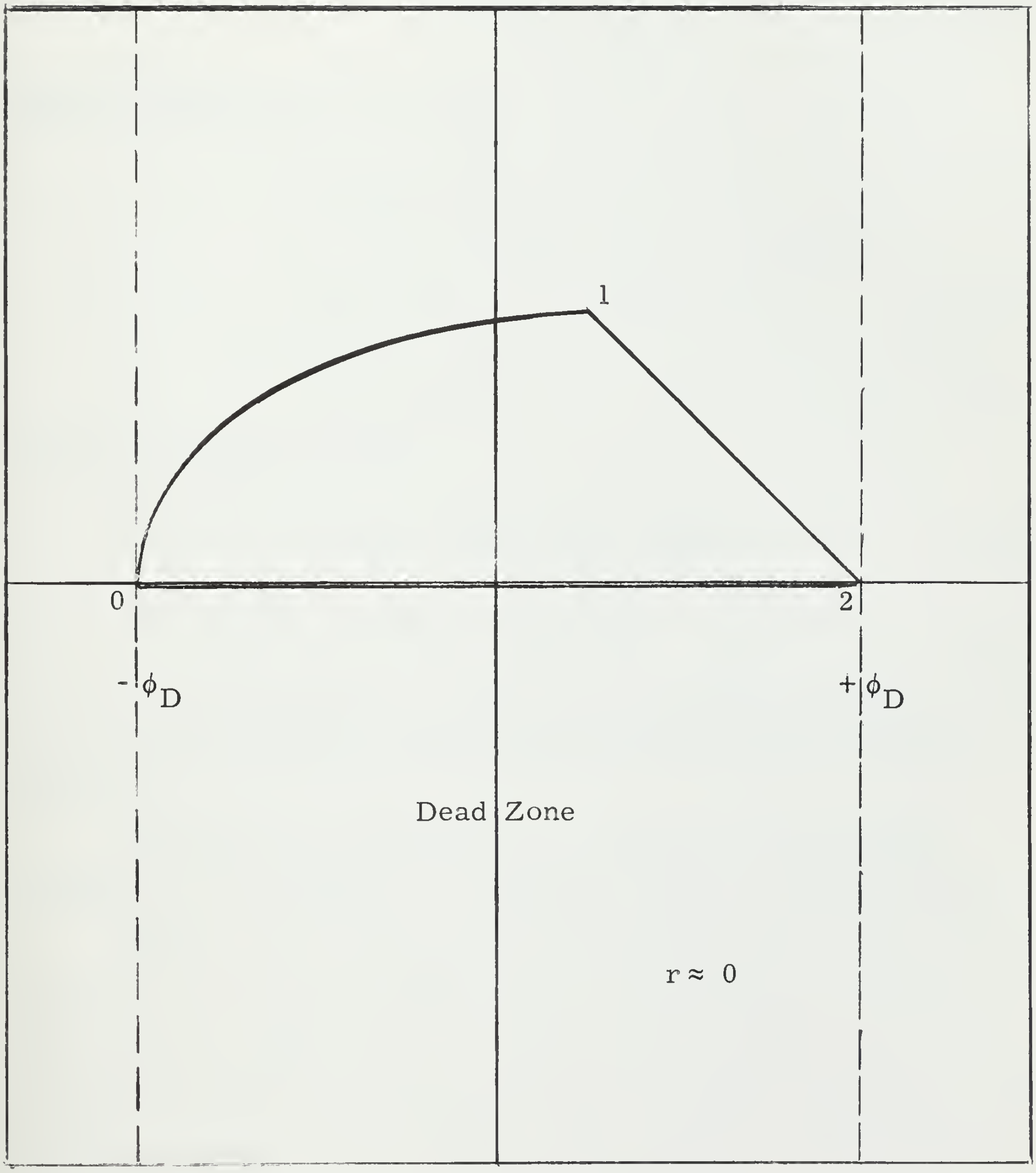


Figure 4-3. Phease Plane Condition for Determining Minimum Width of the Dead Zone in a Compensated Ternary Pulsed Integrating Pendulous Accelerometer.

When the pendulum is coasting, that is, δ equals zero and r equals zero, and the phase plane path has not crossed the zero velocity axis, the motion can be described by

$$\phi + \dot{\phi} = \phi_o + \dot{\phi}_o. \tag{4.8}$$

Hence, when $\dot{\phi}$ equals zero at ϕ_2

$$\phi_2 = \phi_1 + \dot{\phi}_1 \tag{4.9}$$

$$\phi_2 = T_s + e^{-T_s} - 1 - \phi_D + 1 - e^{-T_s} \tag{4.10}$$

$$\phi_2 = T_s - \phi_D. \tag{4.11}$$

To keep ϕ in the dead zone,

$$(+\phi_D) - (-\phi_D) > \phi_2 - (-\phi_D), \tag{4.12}$$

$$(+\phi_D) - (-\phi_D) > T_s, \tag{4.13}$$

or, the non-dimensional magnitude of the minimum width of the dead zone is equal to the magnitude of the non-dimensional sampling period, T_s.

An elemental switching logic is explained and designed in section D. 2 of Appendix D. A symbolic design is shown in Figure D-4.

Figures (4-4), (4-5), and (4-6) show the steady state counting cycles for the alternating pulse zone compensation for values of r in one-tenth increments from 0.1 to 0.9. These values were chosen because the counting cycles are relatively simple yet they exhibit the problems found in the technique. Where practical, two counting cycles are shown and they indicate the boundaries of the area in the phase plane in which the limit cycles can exist.

For the cases of r equal 0.3 and 0.7 only one limit cycle is shown. Inspection of the cycles show they have a very limited freedom of position, ϕ_f. The trajectory of the error on the phase plane will adjust itself and hunt for the limit cycle. In Figure (4-4) the area in

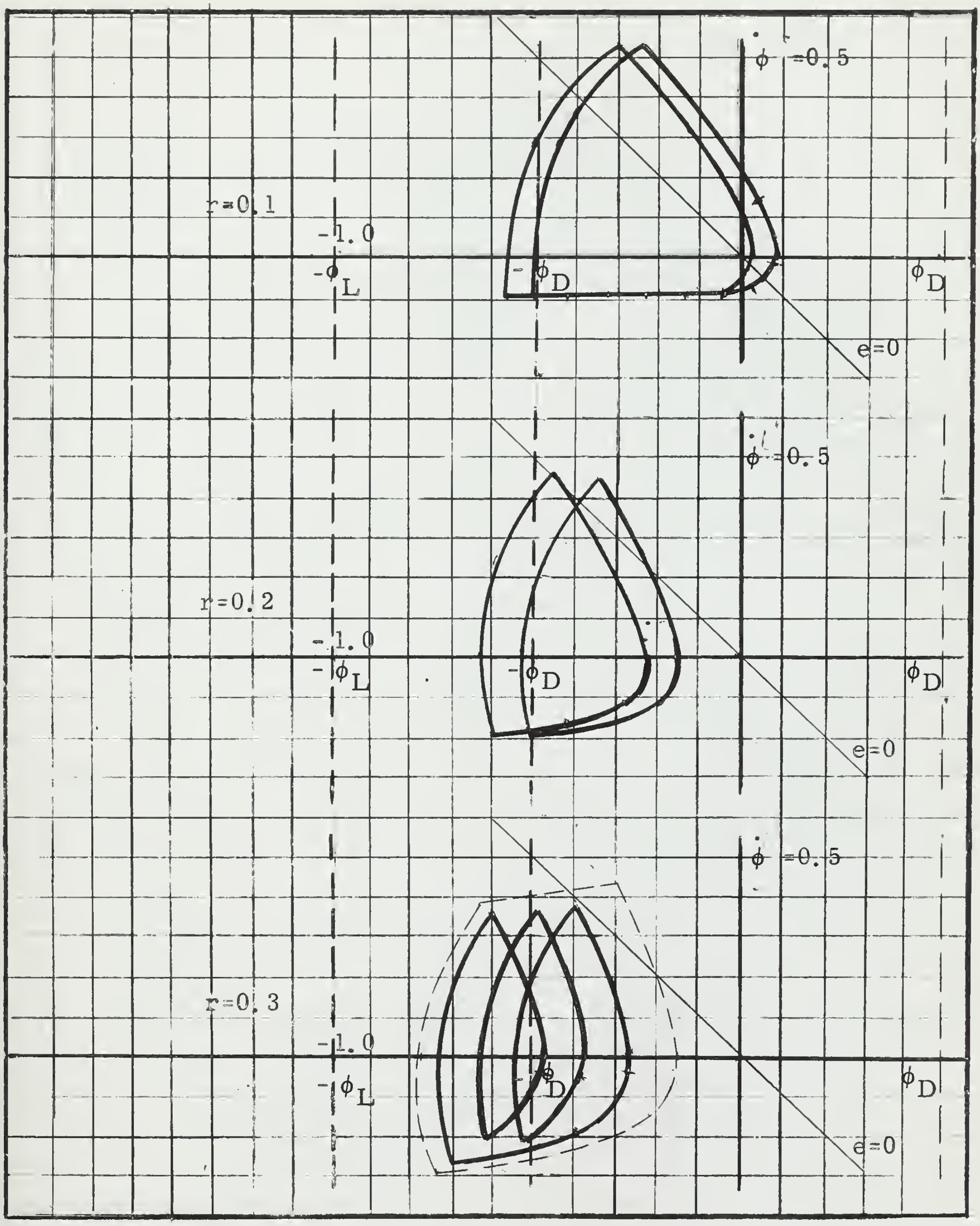


Figure 4-4. Steady State Counting Cycles for Alternating Pulse Zone Compensation, $T_s = 1.0$.

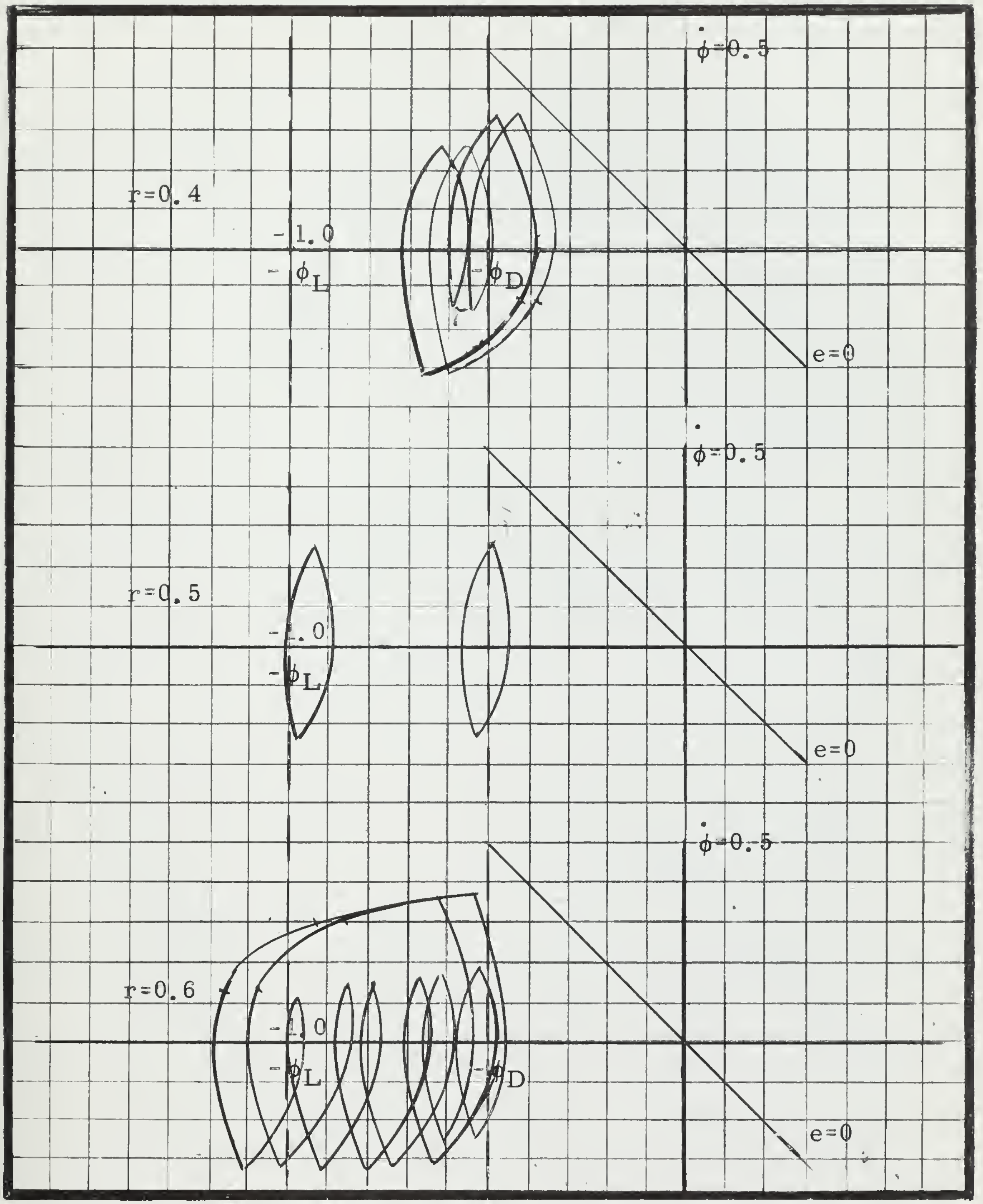


Figure 4-5. Steady State Limit Cycles for Alternating Pulse Zone Compensation, T_s = 1.0.

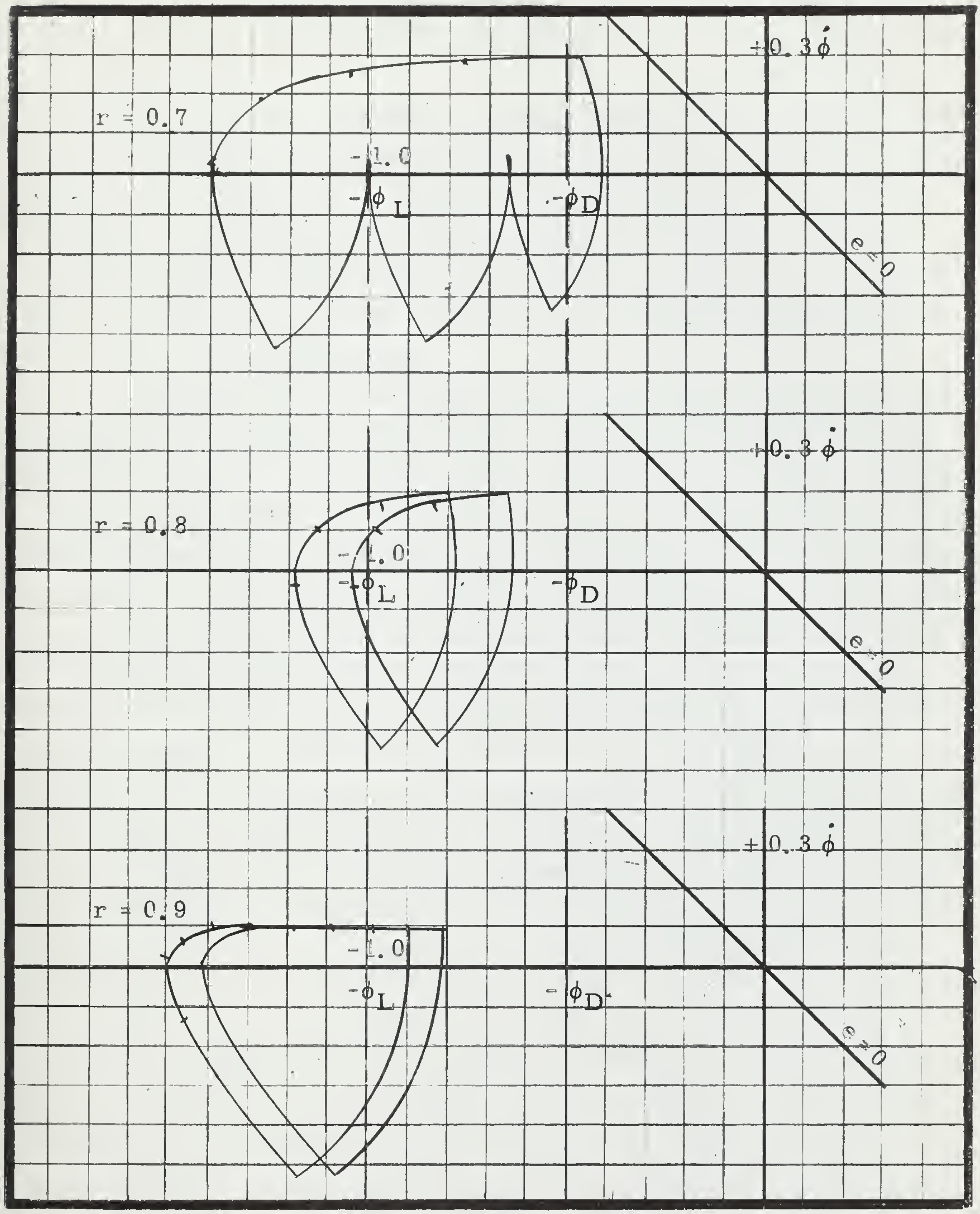


Figure 4-6. Steady State Limit Cycles for Alternating Pulse Zone Compensation $T_s = 1.0$.

which the trajectory for r equal 0.3 hunts, after large transient effects have died out, is shown with dashed lines. One can see that the overall counting error during the hunting period is about that of the steady state limit cycle.

For values of r less than 0.5 the counting cycles are found to be in the lowest order. For values of r greater than 0.5 requiring a two period torque pulse the counting cycles are not in their lowest order. r equal 0.6 requires a torque 2 - not torque 1, torque 3 - not torque 1 counting cycle for lowest order instead of 3 - 1, 1 - 1, 1 - 1, 1 - 1 counting cycle. r equal 0.7 requires 3 - 1, 2 - 1, 2 - 1 counting cycle instead of 5 - 1, 1 - 1, 1 - 1 counting cycle.

In order to have a two period pulse in the alternating pulse zone compensation, the pulse must start in the continuous positive torque zone and move into lead compensation zone before the end of one sampling period in time. For different values of r, meeting the above condition depends on the value of T_s, the non-dimensional sampling time, or the ratio of the sampling period to the time constant of the pendulum.

Figures 4-7 through 4-12 show the steady state counting cycles for alternating pulse zone compensation with T_s equal 5.0. Those values of r which were in their lowest order at T_s equal 1.0 have a larger freedom of position at T_s equal 5.0. For r equal 0.6, Figure 4-10, the lowest order counting cycle can now be assumed with some freedom of position, ϕ_f. For r equal 0.7, Figure 4-9, the lowest order counting cycle is possible, but the system is required to switch at almost a point on the phase plane. The system will "hunt" for this point in the 3 - 1, 1 - 1, 3 - 1 order after large transient effects have died out.

Figure 4-11, for r equal 0.88 and T_s equal 5.0, shows the counting cycle is in its lowest order; 8 - 1, 7 - 1, 7 - 1. At T_s equal 1.0 the ϕ_f is approximately one-fifth that shown in Figure 4-12, again requiring more "hunting" to get in a lowest order counting cycle. For larger values of r which do not require a two sample period pulse in their lowest counting order, the lowest order is possible for small values of T_s, but increasing T_s reduces the amount of "hunting" required to assume the lowest order counting cycle.

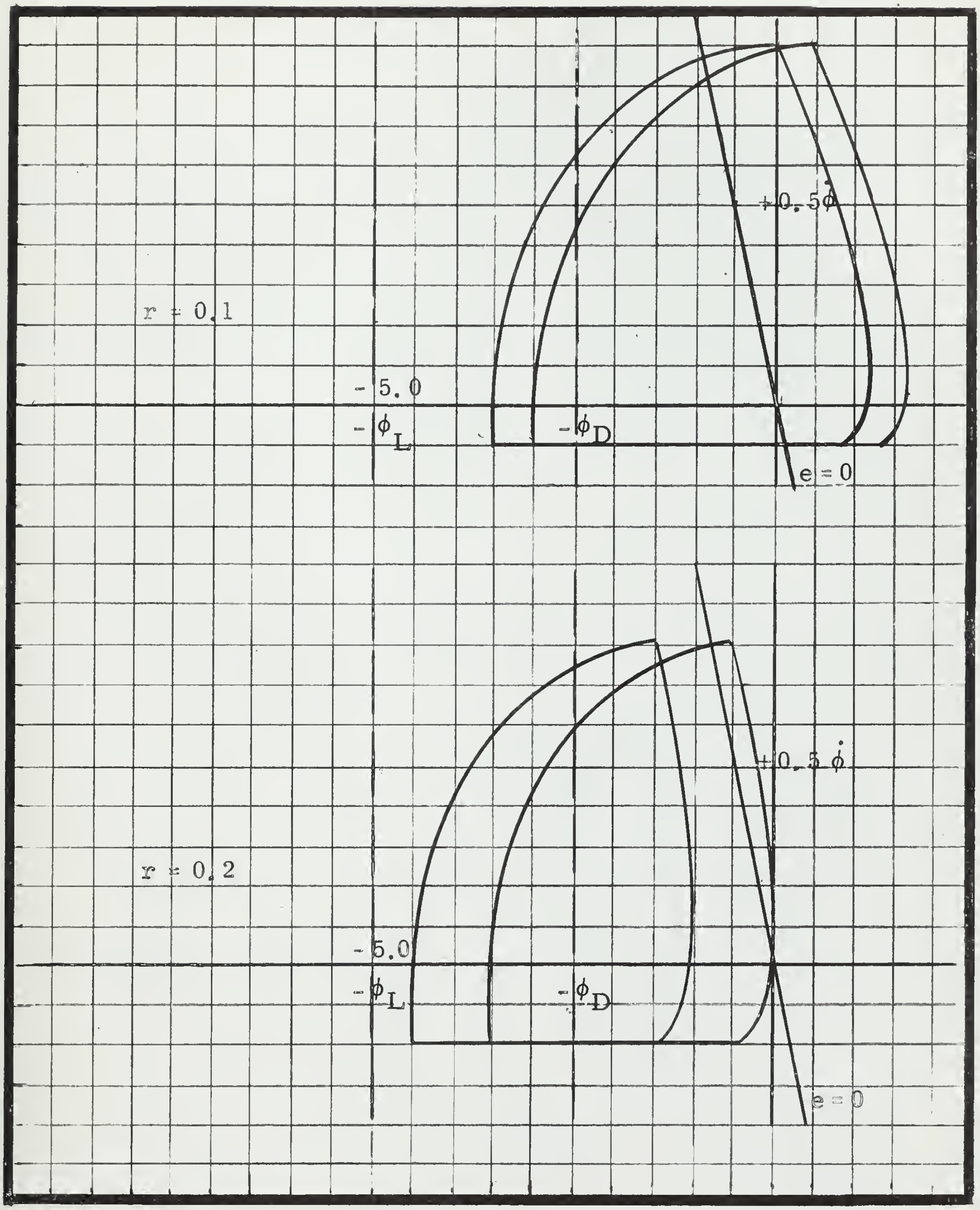


Figure 4-7. Steady State Limit Cycles for Alternating Pulse Zone Compensation $T_s = 5.0$.

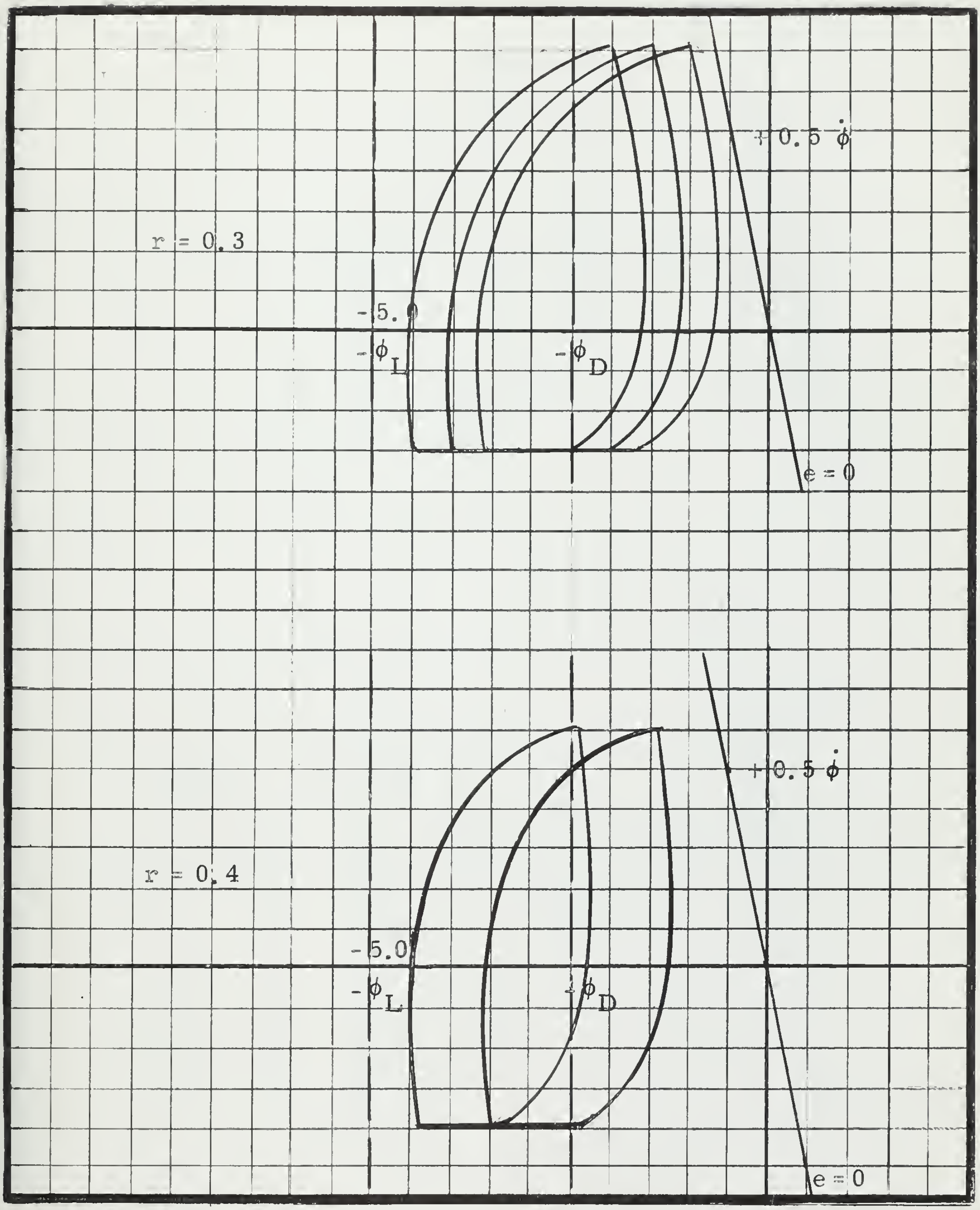


Figure 4-8. Steady State Limit Cycle for Alternating Pulse Zone Compensation $T_s = 5.0$.

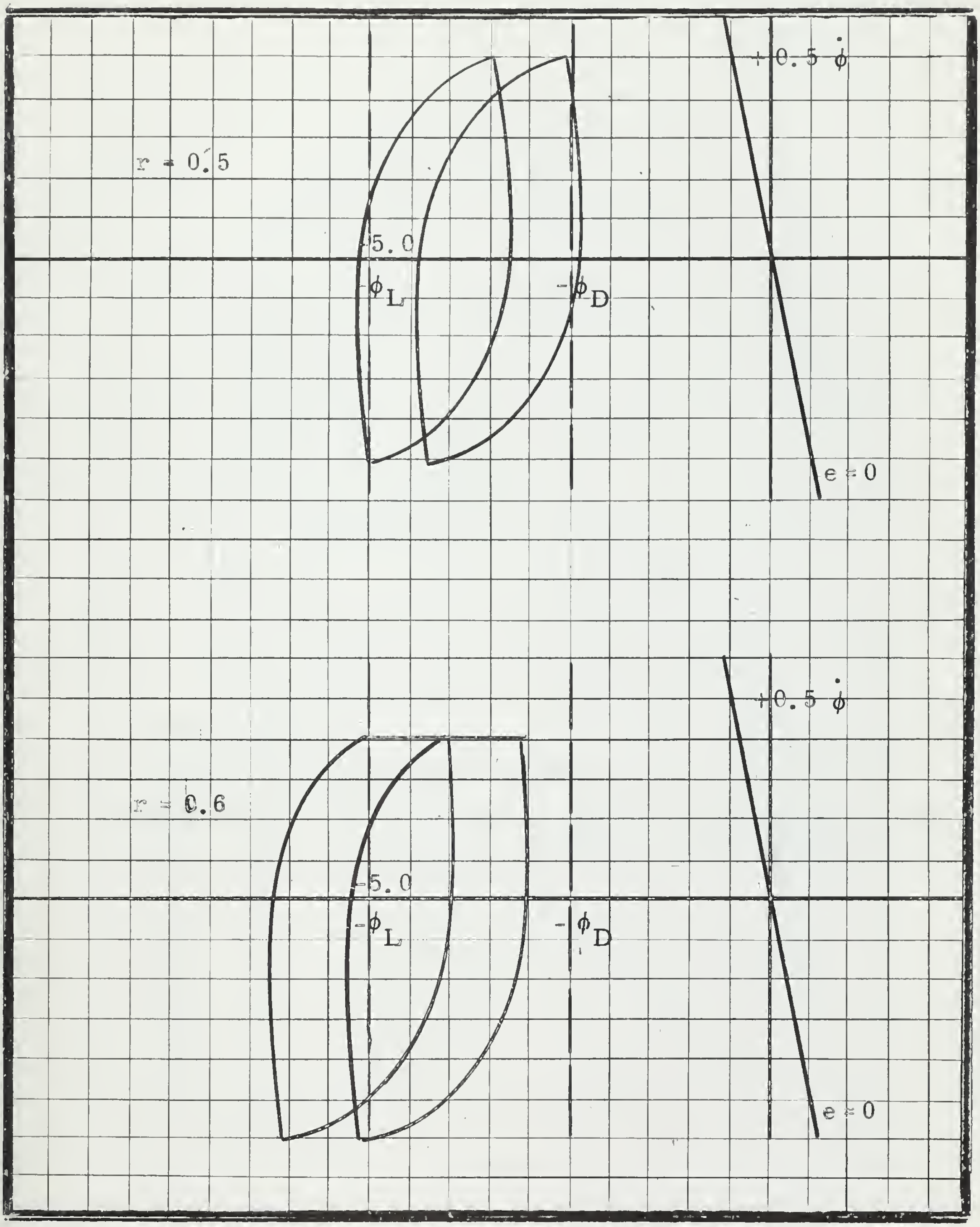


Figure 4-9. Steady State Limit Cycles for Alternating Pulse Zone Compensation $T_S = 5.0$.

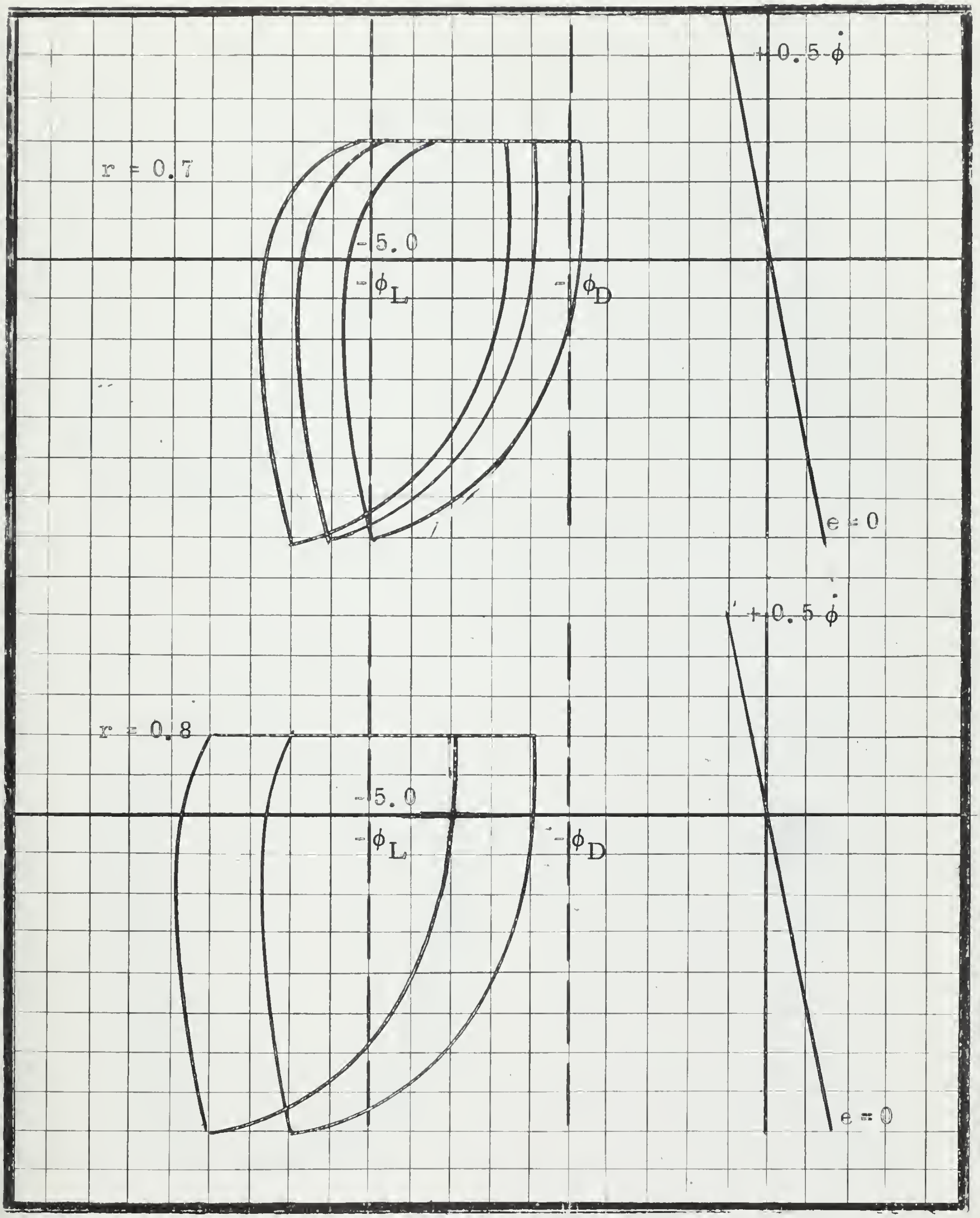


Figure 4-10. Steady State Limit Cycles for Alternating Pulse Zone Compensation $T_s = 5.0$.

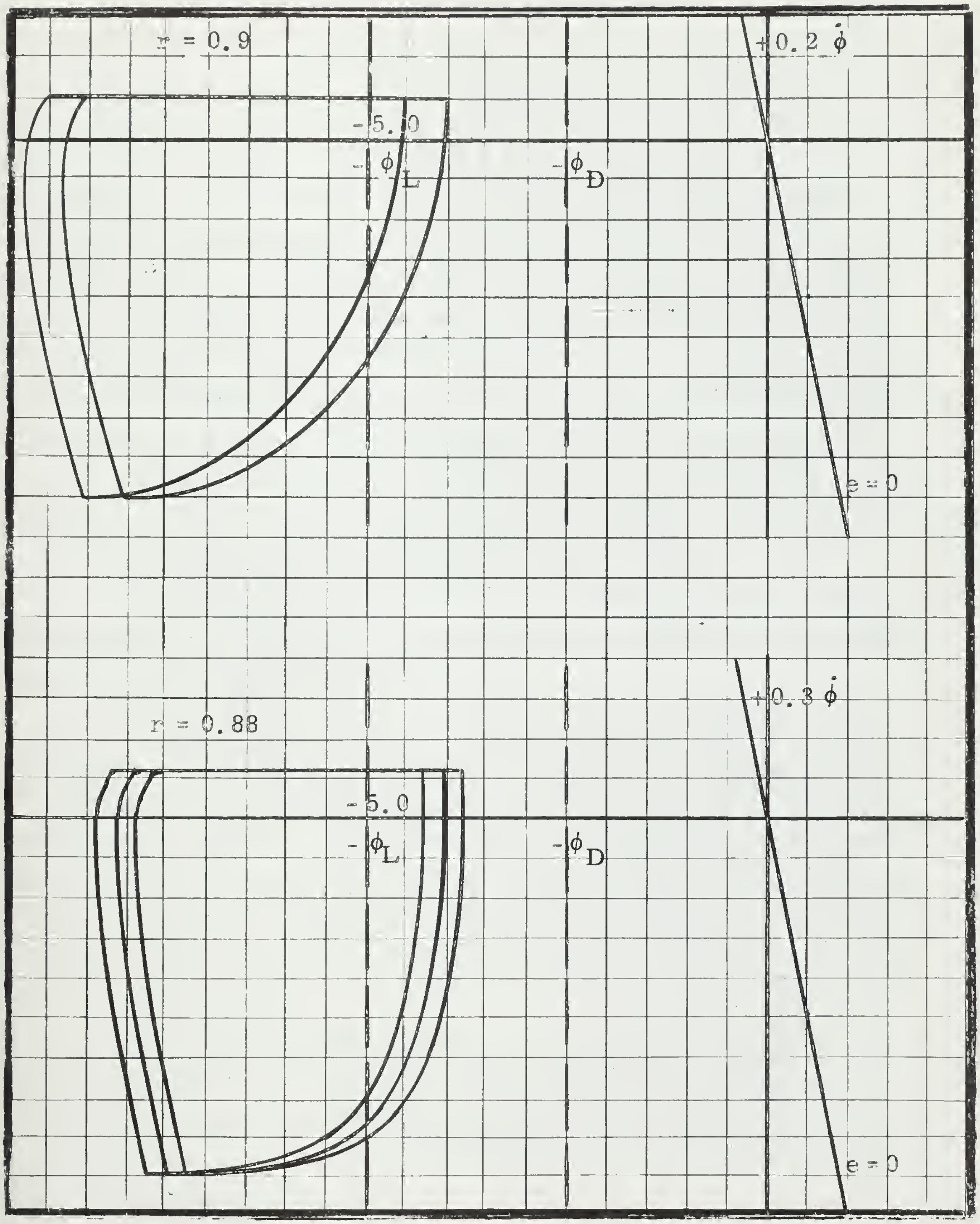


Figure 4-11. Steady State Limit Cycles for Alternating Pulse Zone Compensation $T_s = 5.0$.

4.3. INVERSE ORDER ALTERNATING PULSE ZONE COMPENSATION

In the alternating pulse zone compensation enough lead effect was introduced as the position entered the lead compensation zone from the dead zone. However, on entering the lead compensation zone from the continuous torque zone the system continues torquing one sample period after the first sample in the lead compensation zone before it switched to non-torquing. This removed most of the lead effect. By changing the switching logic so that when a sample is in the lead compensation zone the action during the period following the sampling instant is opposite the action carried out the previous period, that is non-torquing if torqued previous period and vice-versa, an inverse order alternating pulse zone is set up. This will produce the desired lead effect in both the increasing and decreasing ϕ direction.

For the same reasons as in the alternating pulse zone compensation, the width of the dead zone must be greater than T_s and the width of the lead compensation zone greater than $0.5\ T_s$.

The switching logic for the inverse order alternating pulse zone compensation is developed in section D-3 of Appendix D and shown in the diagram of Figure D-5.

The lead effect when switching about the ϕ_D lines was not changed. The phase plane results for values of r from 0 to 0.5 are the same as for the alternating pulse zone compensation shown in Figures 4-4 and 4-5. The phase plane results for T_s equal 1 and r greater than 0.5 are shown in Figure 4-12 and 4-13. The resulting lowest order counting cycles are shown. Also Figure 4-13 shows the expected result, that if T_s is increased the counting cycles remain in their lowest order.

If the compensation problem is viewed from the aspect of not allowing higher order limit cycles by parameter variation as developed in Chapter 3, the same results are obtained as in the inverse order alternating pulse zone compensation except as to the width of the compensation zone.

Referring to Figure C-1, if the portion of the switching line for positive angular rate, $\dot{\phi}$, is moved to the left of its present position,

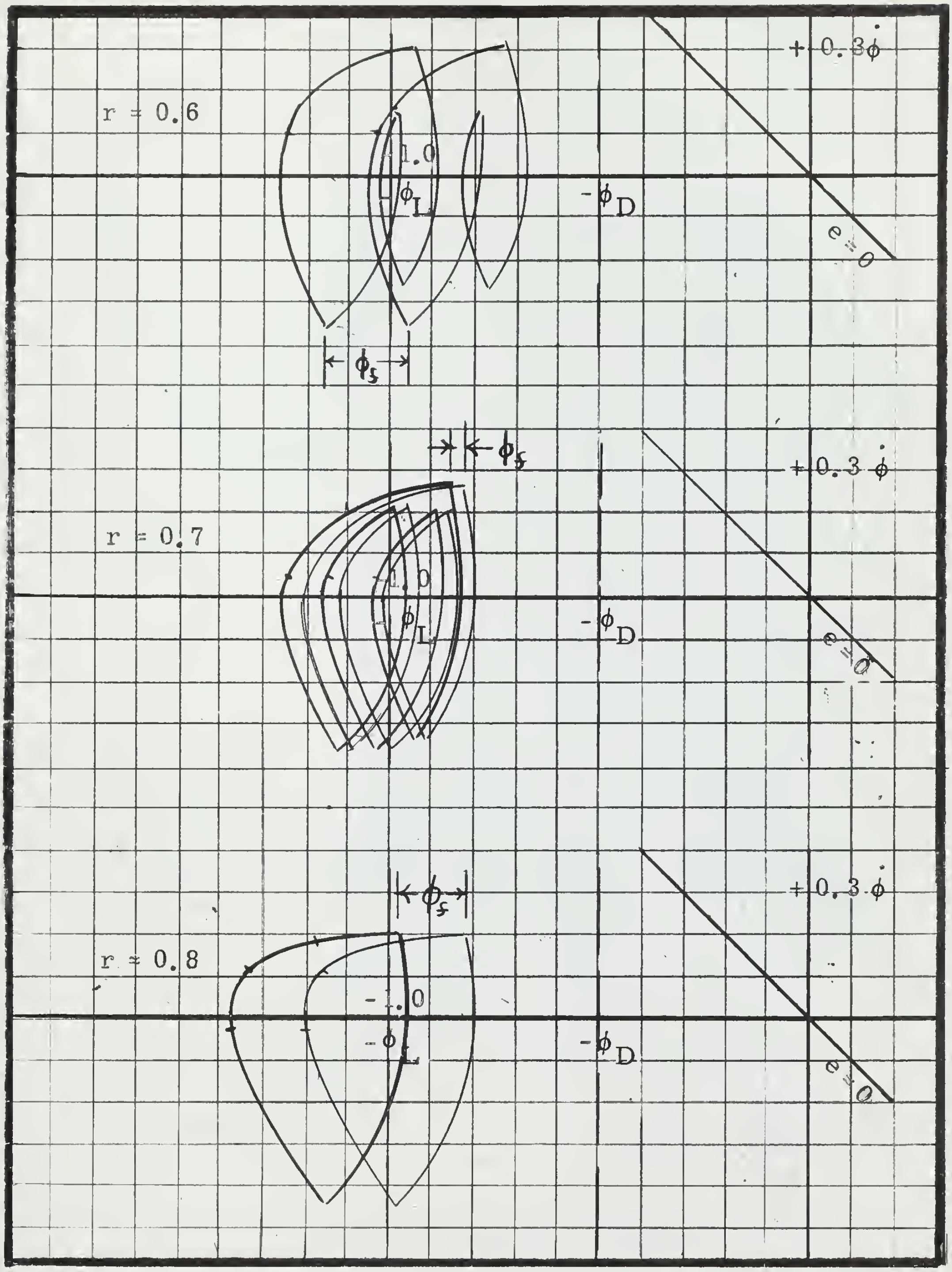


Figure 4-12. Steady State Limit Cycles for Inverse Order Alternating Pulse Zone Compensation $T_s = 1.0$.

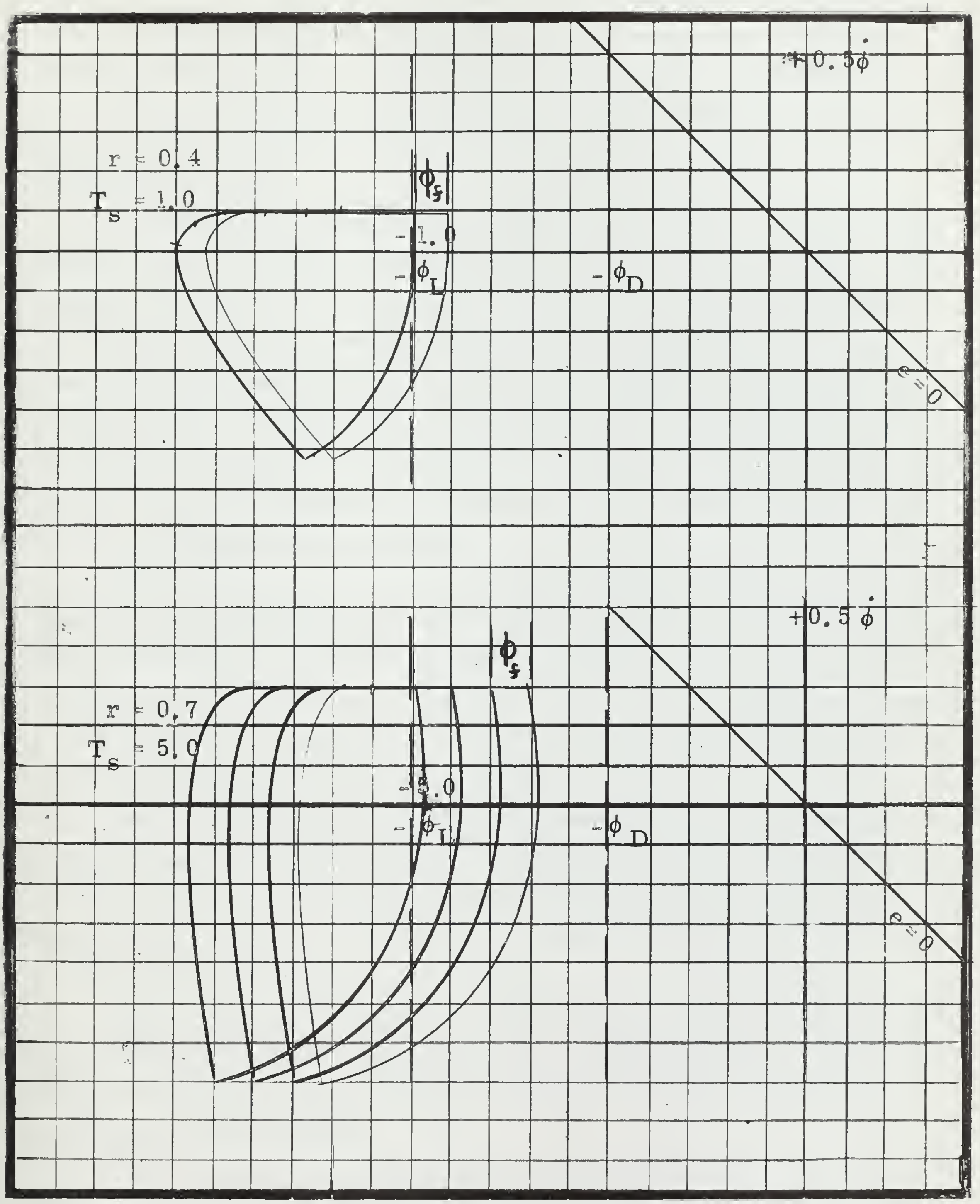


Figure 4-13. Steady State Limit Cycles for Inverse Order Alternating Pulse Zone Compensation.

the N = K - 1 sample will be to the right of the switch line and the system will stop torquing. Similarly if the portion of the switching line for negative angular rate is moved to the right of its present position, the N = K + L - 1 sample will be to the left of the switch line and the system will start to torque. The action described has the effect of making the freedom in positon, ϕ_f, negative for higher order limit cycles.

For the lowest order limit cycles, and r greater than 0.5, the system may non-torque only one sample period, T_s, at a time. The positive $\dot{\phi}$ portion of the switching line can best be instrumented by the positive portion of the ϕ_L switching line of the inverse order alternating compensation zone. By similar reasoning, the negative $\dot{\phi}$ portion of the switching line can best be instrumented by negative portion of the ϕ_D switching line of the inverse order alternating compensation zone. Subsequent samples still in the compensation zone will cause alternating torquing.

From Figure C-2 we see if the switching line separation is made greater than 0.363 for T_s equal 1.0 the 2:2 limit cycle is eliminated. Figures C-3 through C-6 show if ϕ_f eliminates the 2:2 limit cycle, all higher limit cycles are eliminated. $|\phi_L - \phi_D|$ effects ϕ_f. Inspection of Figure C-2 shows that as T_s increases the required ϕ_f does not increase linearly. In fact it does not exceed 1.0.

The logic required is the logic for the inverse order alternating torque zone compensation. Viewing the compensation problem from the parameter variation viewpoint resulted in the same compensation logic but with the minimum width of the dead zone decreased from 0.5 T_s to 0.363 T_s.

The result of decreasing the width of the lead compensation zone is to decrease the error bias and cross coupling effects. But for very large transient situations the time required to reach a steady state limit cycle will be longer.

The phase plane trajectories for T_s equal one are similar to Figures 4-12 and 4-13 except that the $-\phi_L$ line is at -0.363 instead of -0.5 and thus the error bias is reduced by 0.137.

4.4. LOGICALLY FORCED LOWEST ORDER COMPENSATION.

In this compensation, switching logic is devised to allow the system to torque only one period at a time if r is less than 0.5 and to not torque only one period at a time if r is greater than 0.5. Also limit cycle switching will be limited to be about the ϕ_D line in order to reduce cross coupling errors. If r is greater than 0.5 and the system only torques one period at a time the limit cycle position will move toward greater error. If r is less than 0.5 and the system will not torque only one period at a time, the limit cycle position will move to smaller error.

Based on the described movement, when it is desired to switch from logic for r less than 0.5 to logic for r greater than 0.5 the position will be moving toward greater error. If a zone or boundary line called $+\phi_L$ or $-\phi_L$ as appropriate, is set up beyond the greatest error of counting cycles for r less than 0.5, when r is greater than 0.5 the position will increase until it passes this line. When a sample with ϕ greater than ϕ_L is obtained the logic can be changed to torque as required when out of the dead zone, but not torque less than one sampling period, T_s, at a time. The system will then torque until the position returns to the dead zone and then switching will be about the ϕ_D line. This logic system requires the position to return out of the dead zone in only one period of not torquing. To insure this for all combinations of dead zone penetration and all values of r greater than 0.5 it is found that T_s must be at least 5.

The dead zone width requirements are the same as before, $(+\phi_D) - (-\phi)$ greater than T_s.

Then with T_s equal 5 or greater, r will be less than 0.5 when two consecutive samples are found to be in the dead zone. This can be used to change the switching logic when r becomes less than 0.5. For consecutive samples in the zone between ϕ_L and ϕ_D, the system will torque alternating periods.

The logic for this compensation is developed in section D-4 and shown in Figure D-6.

Figures 4-14 through 4-18 show the counting cycle for the logically forced lowest order compensation to be in the lowest order.

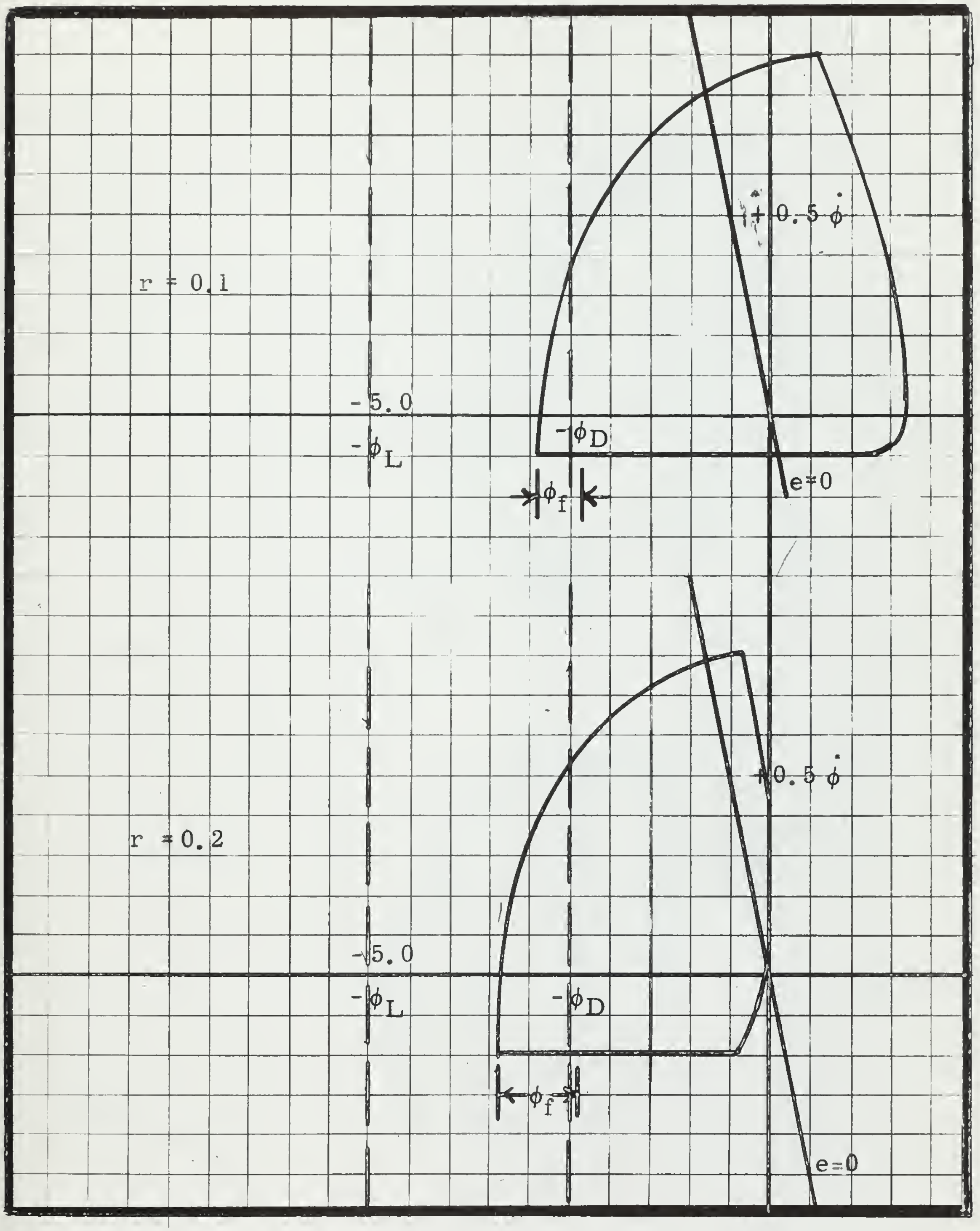


Figure 4-14. Steady State Limit Cycles for Logically Forced Lowest Order, $T_s = 5.0$.

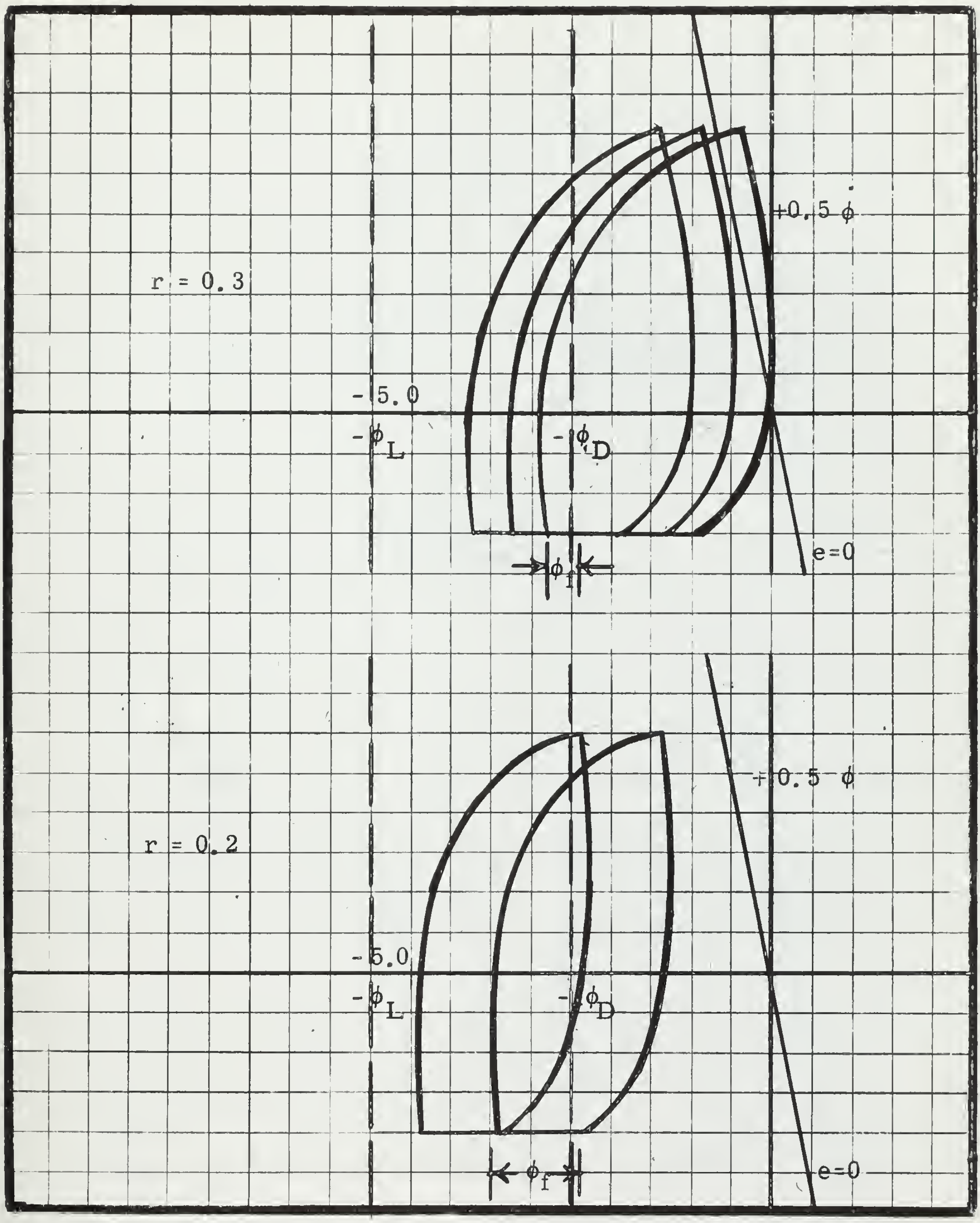


Figure 4-15. Steady State Limit Cycles for Logically Forced Lowest Order, $T_s = 5.0$.

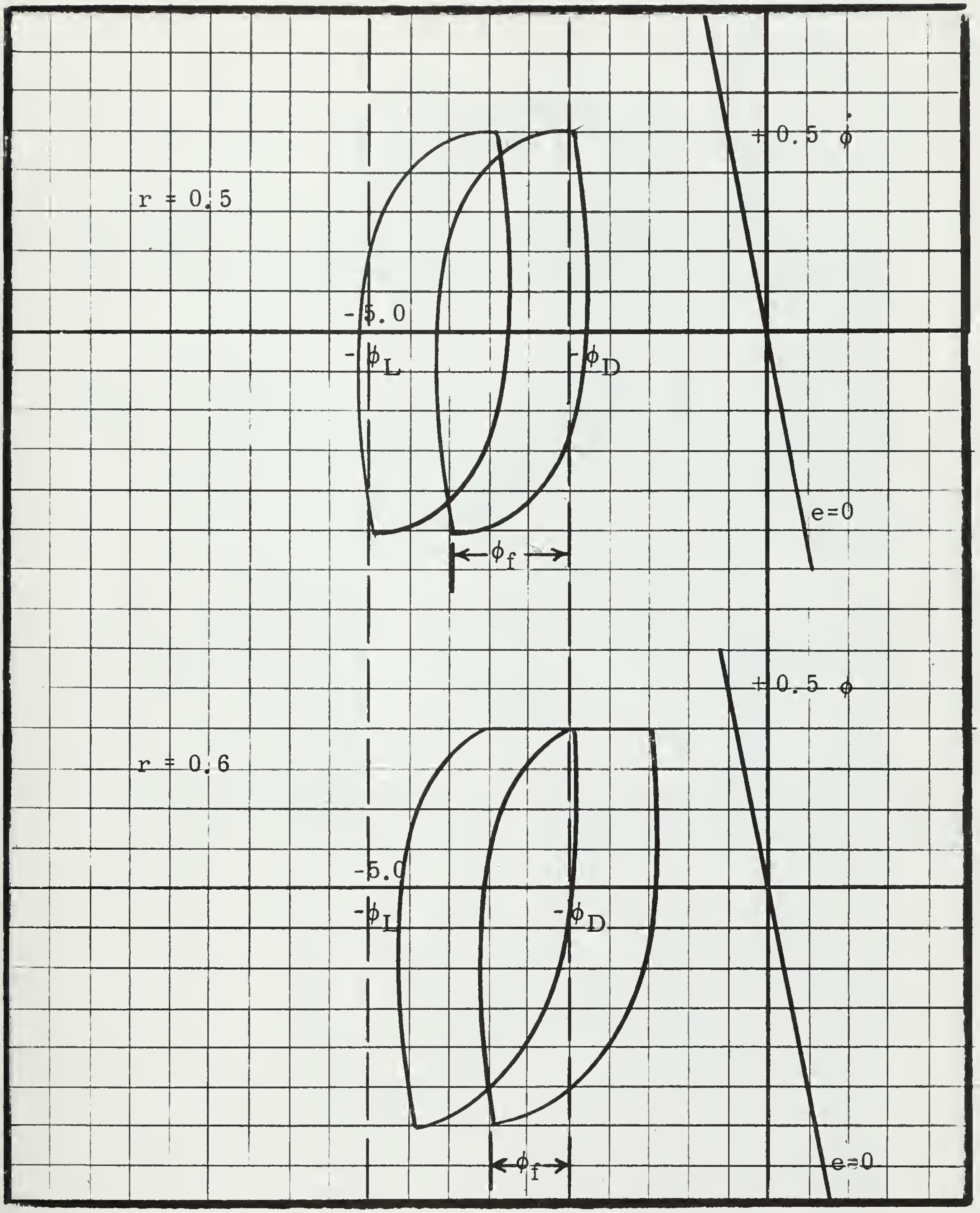


Figure 4-16. Steady State Limit Cycles for Logically Forced Lowest Order, $T_s = 5.0$.

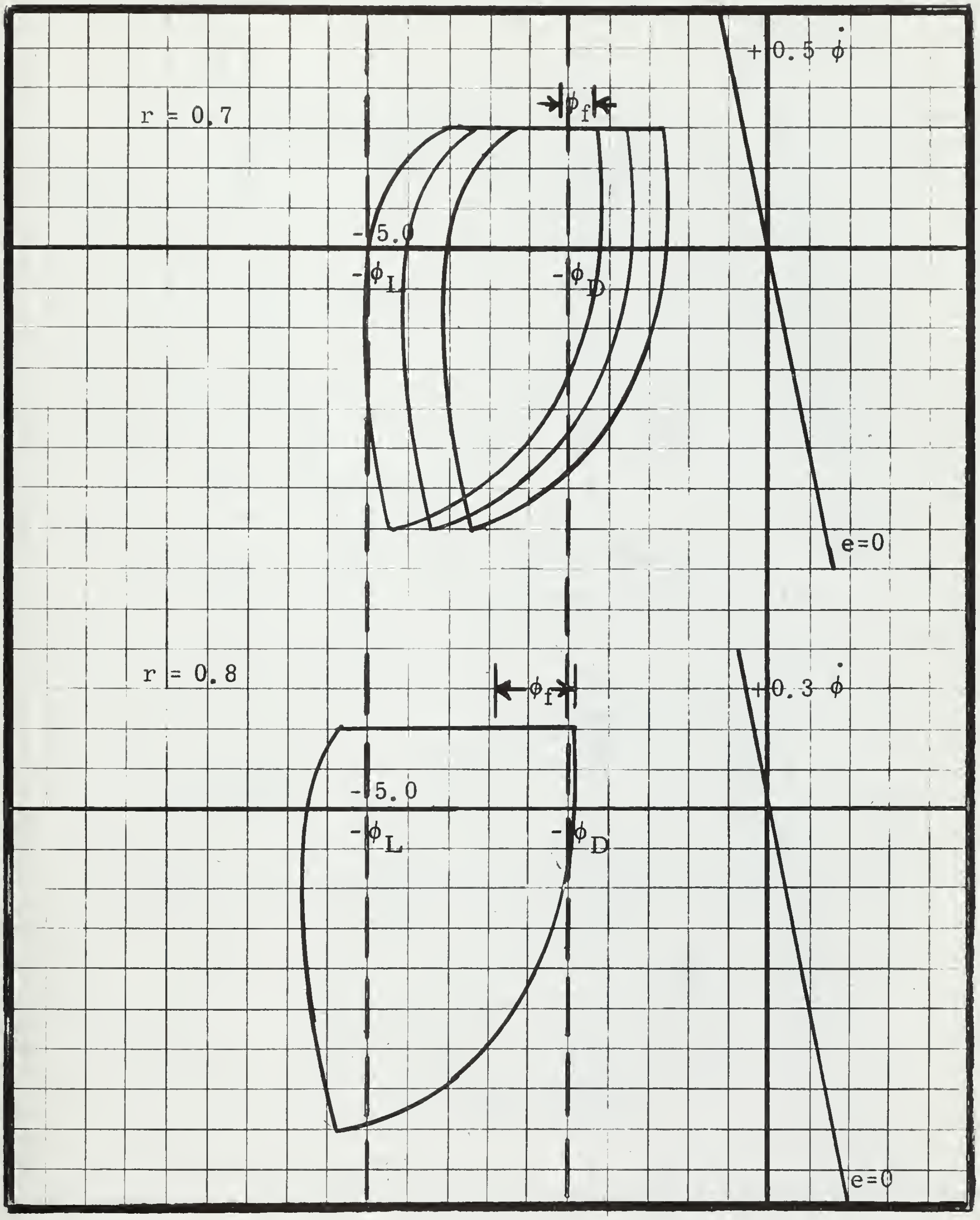


Figure 4-17. Steady State Limit Cycles for Logically Forced Lowest Order, T_s = 5.0.

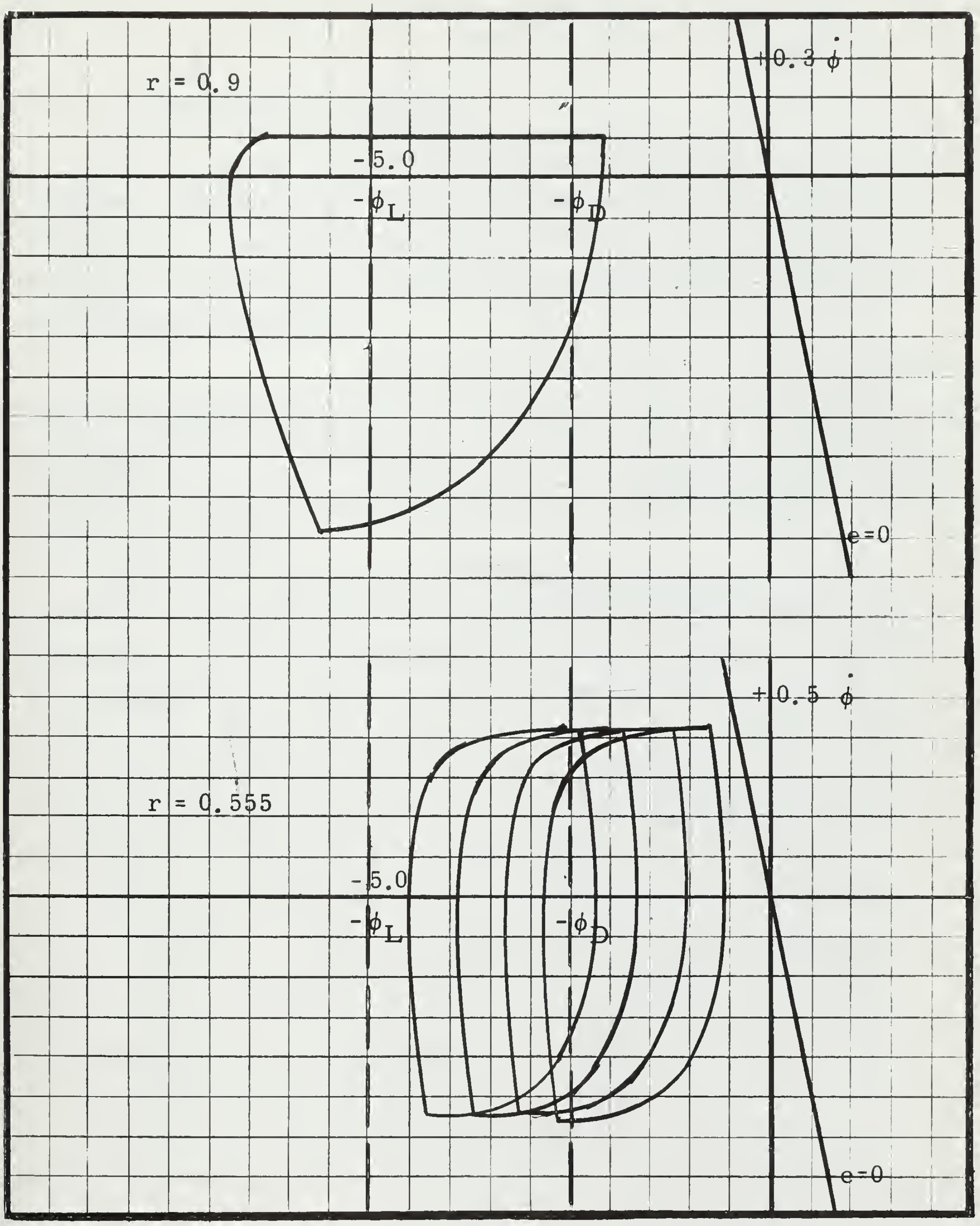


Figure 4-18. Steady State Limit Cycles for Logically Forced Lowest Order, T_s = 5.0.

In fact for values of r other than in the 0.5 vicinity the counting cycles are exactly the same as in the non-compensated system with T_s equal 5.0. Only in the vicinity of 0.5 is the counting cycle improved.

4.5. COMPENSATION RESULTS

On each of the figures showing limit cycles the zero error line is shown. This allows one at a glance to get a relative estimate of the error size. In Figure 4-19 through 4-24 the values of the error during the limit cycles are shown. The errors were computed using the previously developed

$$E = \phi + \dot{\phi} \ . \tag{4.14}$$

E is the non-dimensional error expressed in counts of velocity quanta. The largest error allowed by the freedom of position is shown. The following conclusion can be drawn from the error plots and the phase plane trajectories.

In the alternating pulse zone compensation T_s must be increased to 5 to insure the counting cycles are in their lowest order. The only performance improvement is in the vicinity of r equal 0.5. The hardware restrictions placed on the system by the uncompensated system due to a large T_s has not been removed. In addition, for values of r greater than 0.5 the error bias has doubled. The more complicated error detection and switching logic is a relatively high price to pay for doubtful performance improvement.

The logically forced lowest order compensation also requires T_s greater than 5. It has the same error bias as the uncompensated system. Its counting cycles are the same as those of the uncompensated system with T_s equal 5 except when r is in the vicinity of 0.5. The logically forced lowest order compensation does produce the lowest order counting cycles for all values of r.

The inverse order alternating pulse zone compensation produces the lowest order counting cycle for all values of r with T_s equal one or greater. For r equal 0.5 and greater the error bias is greater than in an uncompensated system due to the switching about the ϕ_L line. The switching logic shown in Appendix D for the inverse alternating pulse zone compensation is relatively simple.

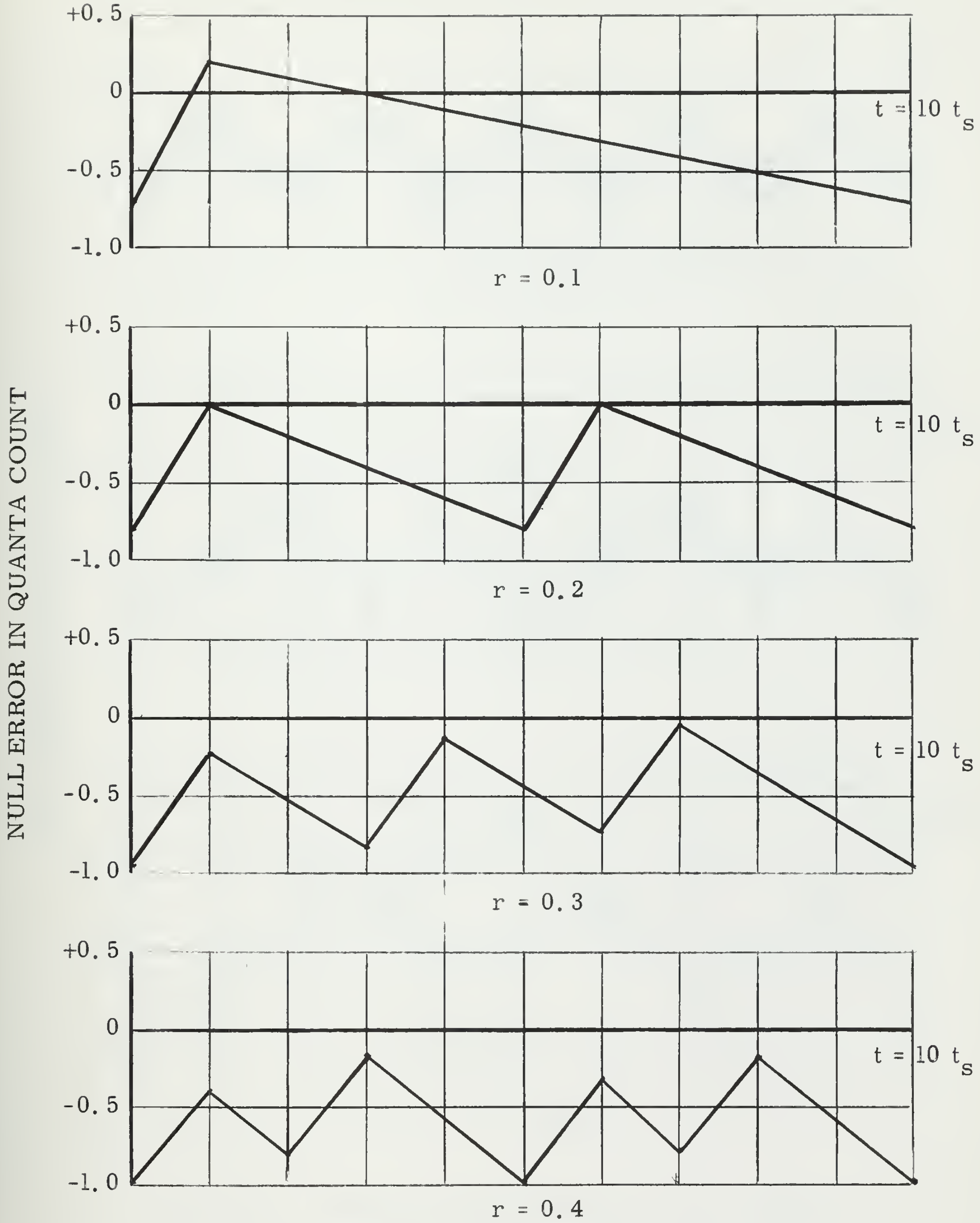


Figure 4-19. Error Cycles for Compensated Systems With T_s = 1.0

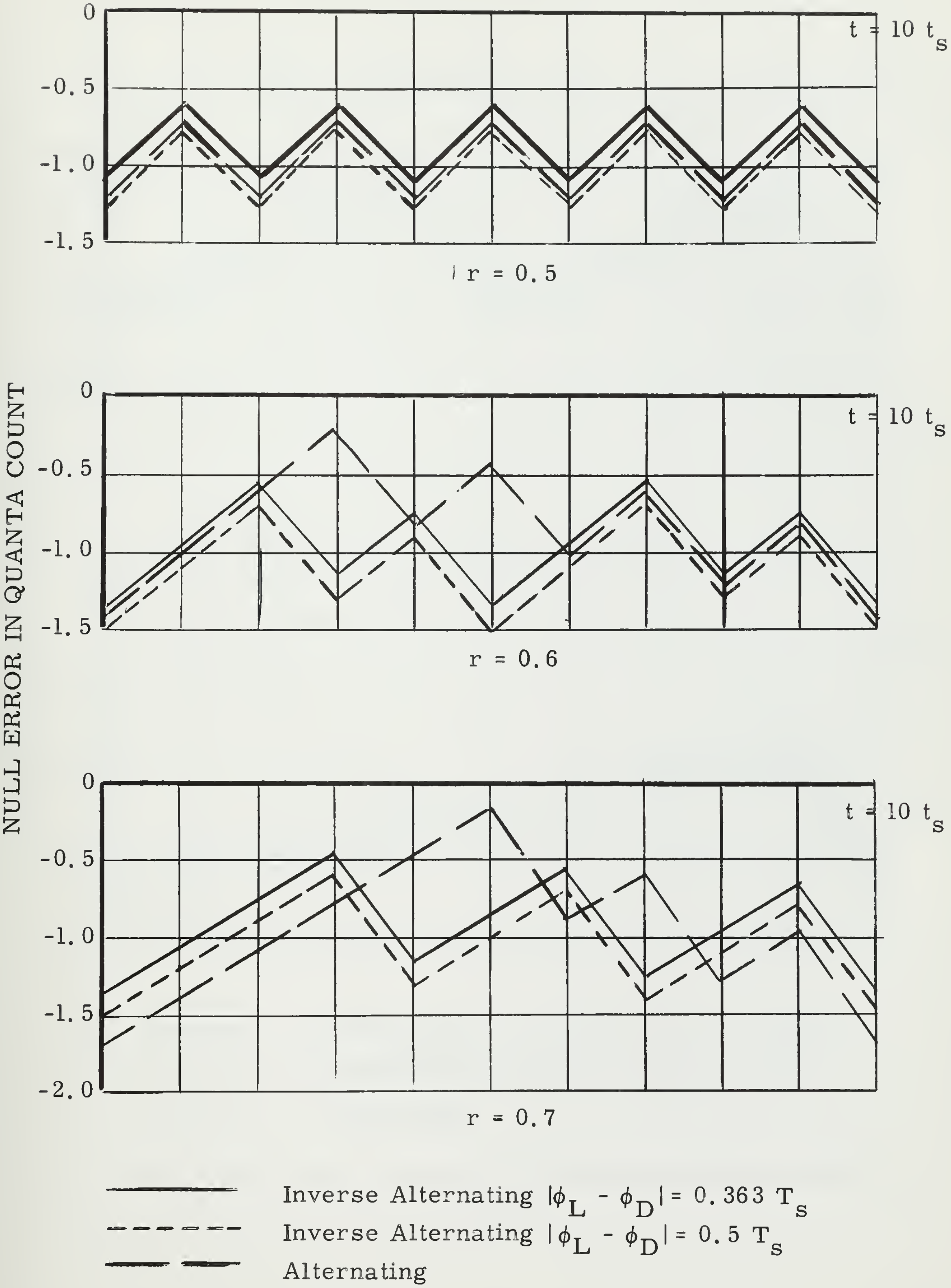


Figure 4-20. Error Cycles for Compensated Systems With $T_s = 1.0$.

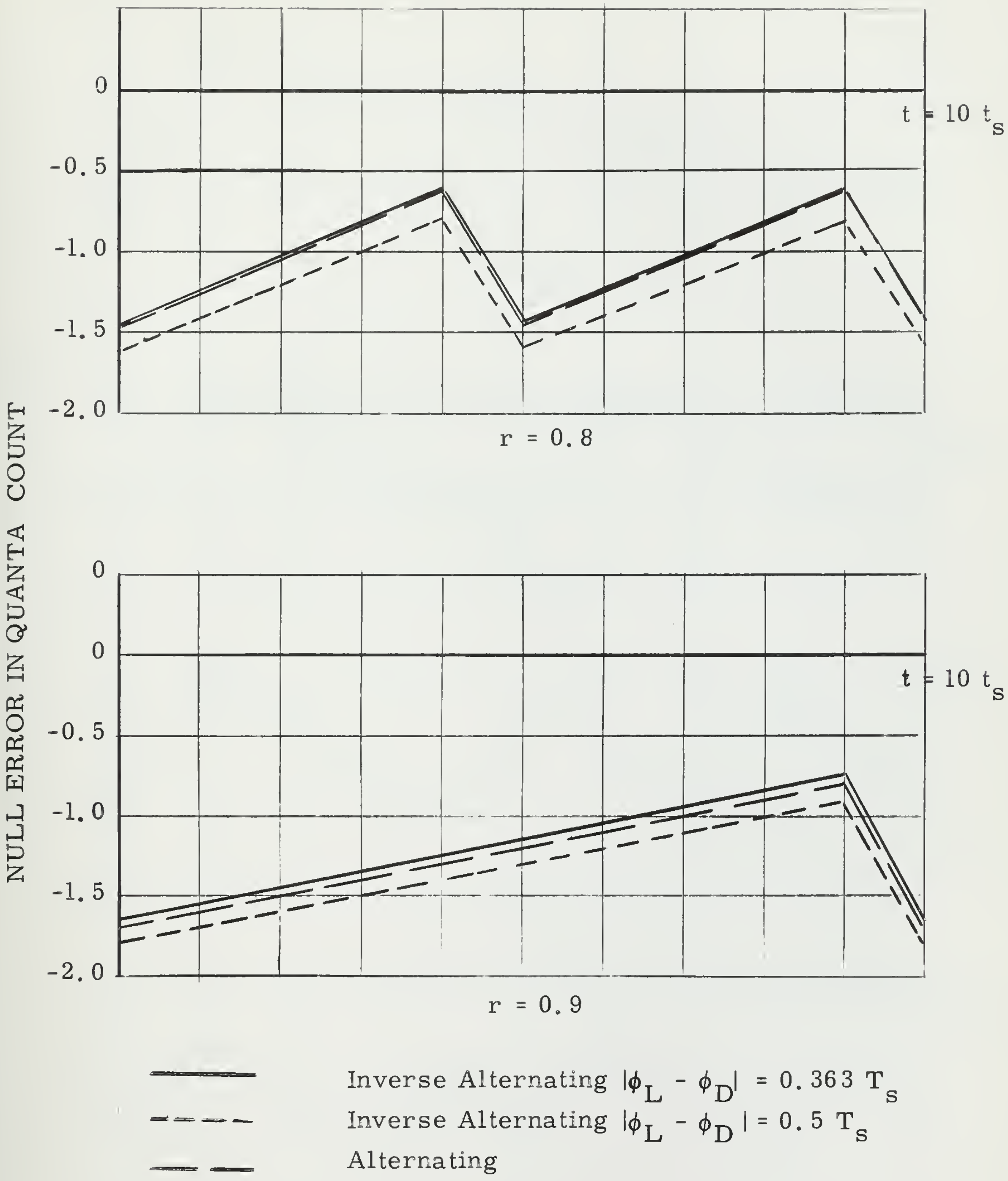


Figure 4-21. Error Cycles for Compensated Systems With $T_s = 1.0$.

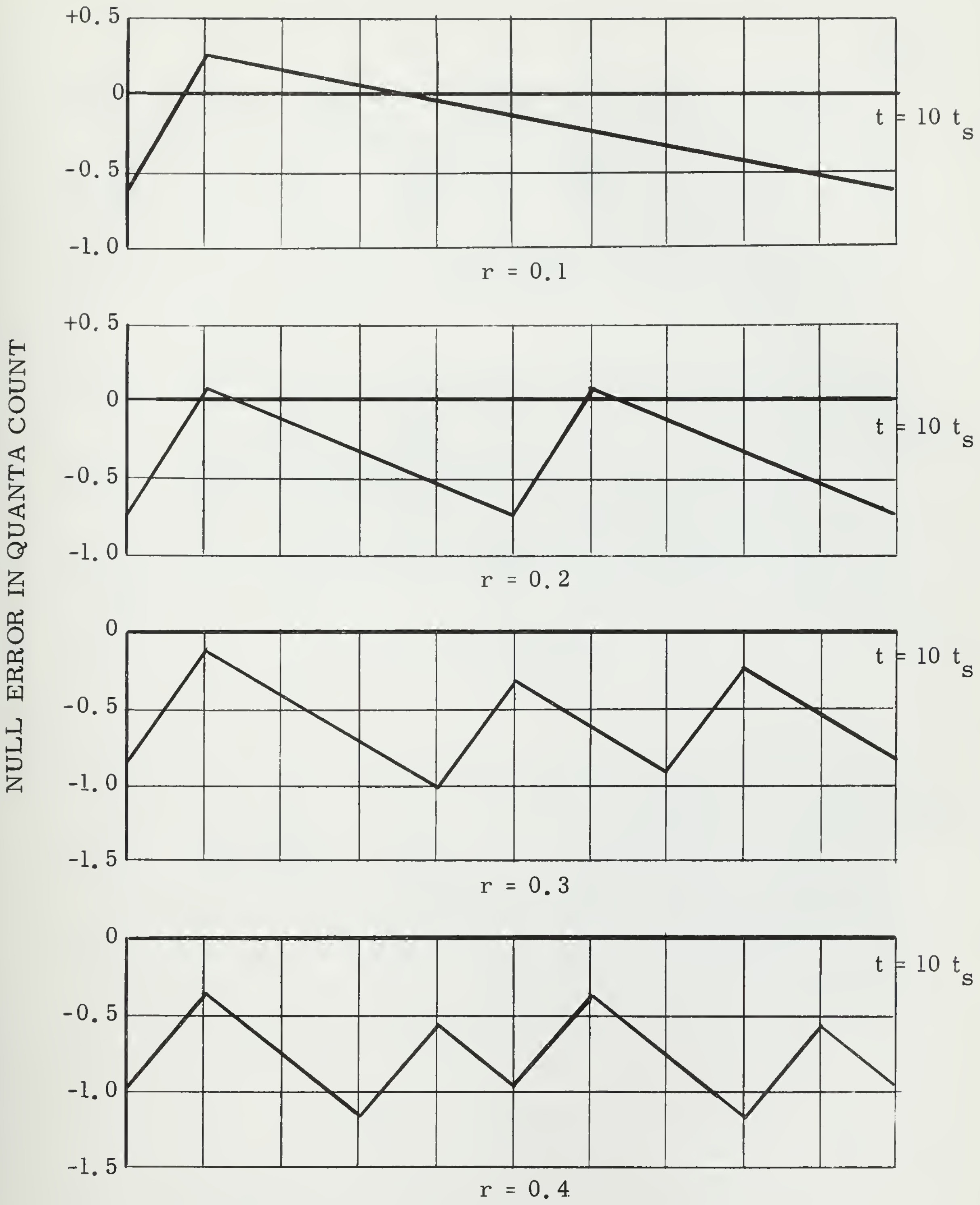


Figure 4-22. Error in Compensated Systems With T_s = 5.0

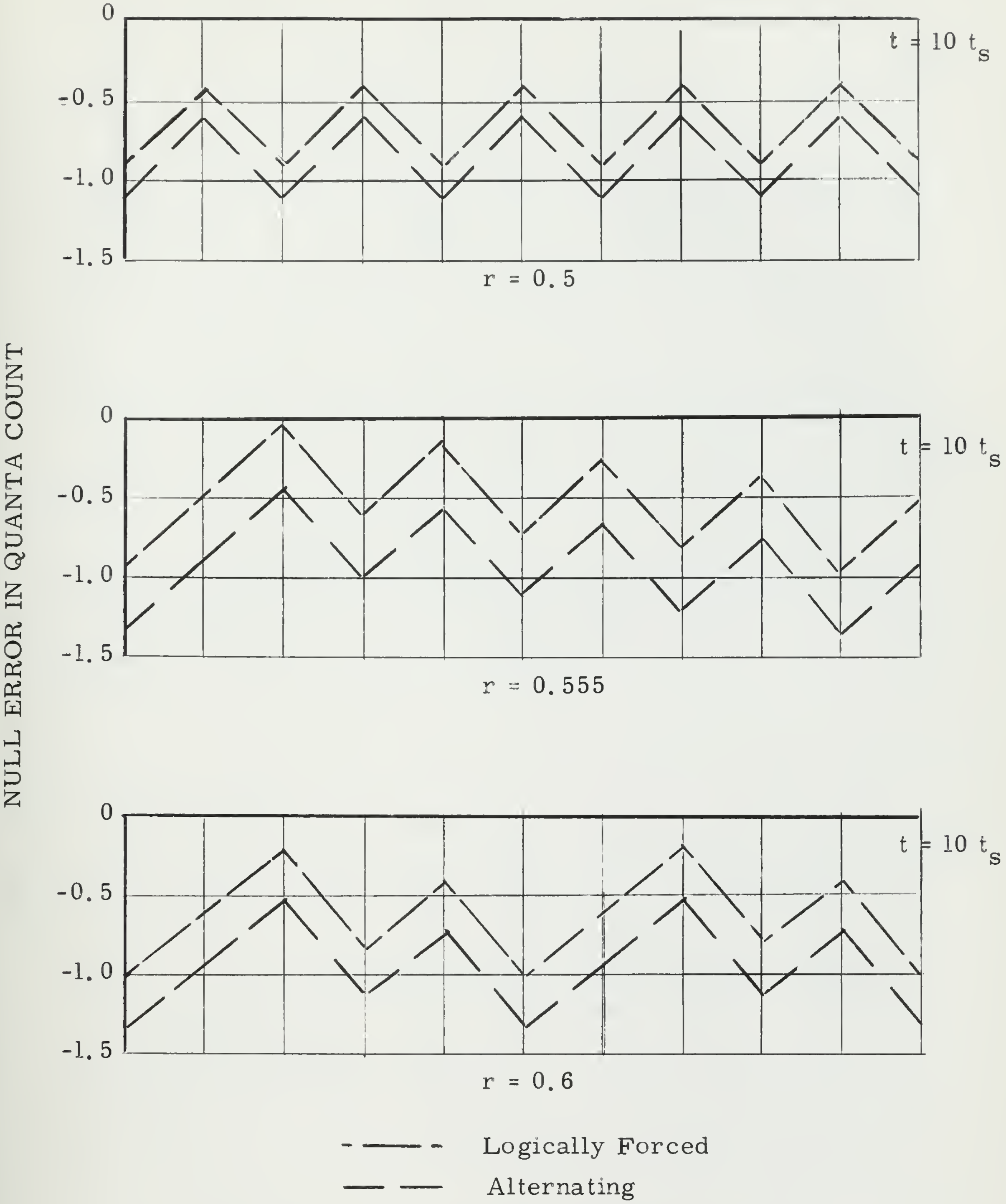


Figure 4-23. Error in Compensated Systems With T_s = 5.0.

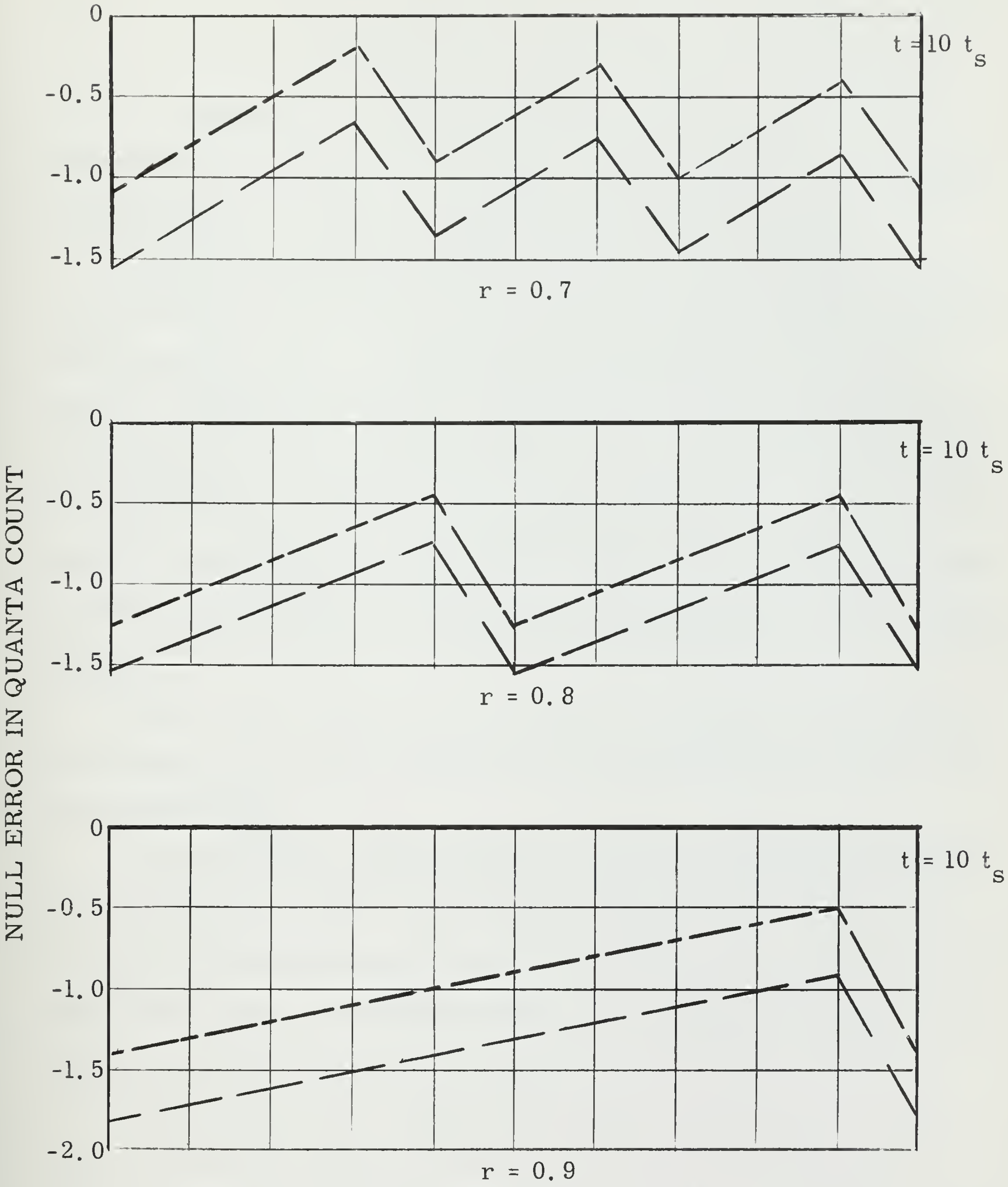


Figure 4-24. Error in Compensated Systems With $T_s = 5.0$.

The velocity quanta size, the maximum acceleration to be measured, and the computer clock frequency usually are fixed by the application of the pulsed pendulous accelerometer. The dead zone of the error detector is fixed in minimum size. These fixed parameters, with the additional requirement that the width of the dead zone be greater than one T_s, place stringent demands on the ratio of the pendulum moment of inertia J to the damping constant C. Smaller minimum required T_s eases the restriction on the J to C ratio.

With the non-dimensional width of the dead zone equal to one T_s, the error bias due to switching about the ϕ_D line is one-half a velocity quanta and that due to switching about the ϕ_L line is one velocity quanta. Sections D. 5 and D. 6 of Appendix D develop logic to correct the velocity count for the error bias due to switching about particular switching lines. This correction makes it practical to make the dead zone more than one T_s wide in non-dimensional magnitude. This will ease the restrictions put on the J to C ratio. Correcting the velocity count for the error bias does not decrease the physical angle the pendulum makes with its null position. It will not decrease cross coupling effects.

With proper error bias correction and using the inverse order alternating pulse zone compensation, the counting error can be kept within one quanta.

In describing the limitations of the error detector the conservative statement was made that the best magnitude resolution to be expected was about equal to the width of the dead zone. This assumption was based on the high noise to signal ratio due to a relatively low error information rate. If quadrature effects about the microsyn null position are the major factor in creating the error detector's dead zone size, better resolution away from the dead zone can be expected.

Better resolution away from the dead zone will allow the non-dimensional dead zone to be several T_s wide, but the distance between ϕ_L and ϕ_D still only needs to be 0.5 T_s. ϕ_L can then be moved in towards the dead zone decreasing cross coupling effects. This will require error count correction because the dead zone will become the several counts wide. Also, because the dead zone is several

counts wide the random initial error increases. This could be prevented by electrically restraining the pendulum until the count is initiated. This would require a special counting error bias initial correction

4.6. COMPENSATION SUMMARY

Three logic compensation schemes were devised. The alternating pulse zone compensation with a small amount of switching logic traded larger error bias for keeping the counting cycle in its lowest mode for all values of r. It required T_s equal to five or more placing design restrictions on the hardware. The logically forced lowest order did not increase the error bias in making all counting cycles the lowest order, but it again requires T_s equal five or more and uses more complicated switching logic. The inverse order alternating pulse zone compensation puts the counting cycles in their lowest order and only requires T_s equal to or greater than one. The switching logic is simple. For values of r greater than 0.5 the error bias is increased. Velocity count correction schemes were devised to correct the counted velocity for the existing error bias. The inverse order alternating pulse zone compensation is the most practical because it does not place stringent moment of inertia - damping coefficient ratio restrictions on the pendulum unit.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1. CONCLUSIONS

The theoretical feasibility of ternary pulse torquing a pendulous accelerometer has been shown. The inherent greater stability of a ternary system as compared to a binary system results in smaller limit cycles associated with smaller velocity quanta counting error.

By adjusting the ternary pulsed accelerometer system's components so as to make the non-dimensional sample time, T_s, 5.0 or greater, the lowest order limit cycles can be assured. When practical considerations do not allow adequate adjustment, switching logic compensation can be used to insure the lowest order limit cycles with T_s equal or greater than 1.0. In compensating the ternary mode pulsed pendulous accelerometer increased error bias is traded for smaller limit cycles. If the error bias is too large, the velocity quanta count in the computer can be corrected for the error bias. Another result of the compensation is increased cross-coupling effects. These cannot be as easily corrected in the velocity count and thus must remain small.

Ternary mode of pulse torquing pendulous integrating accelerometers allows increased accelerometer performance to be "bought" with increased system complexity.

5.2. COMBINATION SYSTEM

Experience gained in studying the ternary mode pulse torqued pendulous integrating accelerometer leads to the consideration of a combination system. Ternary in mode of error detection, that is, the system senses clockwise error, dead zone, and counter-clockwise error; and binary in mode of torquing, that is, torque clockwise or counter-clockwise. The ternary mode in error detection would be used to effect lead in the torque switching. The sense of torquing would be changed on <u>entering</u> the dead zone. Investigation to determine if enough lead would be introduced to insure lowest order limit cycles is recommended.

5.3. RECOMMENDATIONS FOR FURTHER INVESTIGATION

The hardware problems specific to ternary pulse torquing have not been investigated. Investigation to determine the feasibility of having a system produce a reproducible torque pulse in two senses and also not produce it for specific periods needs to be carried out. An energy sink might be the solution except in the case when the ternary mode is used to measure small accelerations at little expenditure of energy. The environmental effect of a varying rate of energy expenditure by the torquer should be studied. The exact performance of error detectors at high sampling rates both in the dead zone region and away from the dead zone is needed to insure the feasibility of the compensation schemes.

Investigation in the use of ternary mode pulse torquing gyros, gimbals, and other inertial instruments is recommended.

APPENDIX A

NOTATION

A_{Δ}	Dead zone of error detector
A	Angular Position of the pendulum with respect to the null position
a	Component of acceleration along the input axis of the accelerometer
a_{RA}	Component of acceleration along the Reference Axis (or null axis) of the accelerometer
C	Coefficient of viscous friction
J	Moment of inertia of the pendulum
t	Time
t_s	Time between samples
M_{tg}	Torque Generator torque
M	Magnitude of Torque Generator torque
P	Pendulosity of the Pendulum
v	Velocity increase
v_i	Indicated velocity
e	Total error in velocity indication
e_d	Digital error
e_n	Null error
e_c	Cross-coupling error
δ	Torquing mode, +1, 0, or -1.
ϕ	Non-dimensional null angle
τ	Non-dimensional time
$\dot{\phi}$	$d\phi/d\tau$
ϕ_{Δ}	Non-dimensional dead zone
r	Non-dimensional acceleration along input axis
r_{RA}	Non-dimensional acceleration along reference axis
V	Non-dimensional velocity
E	Non-dimensional velocity error
E_n	Non-dimensional null error
E_c	Non-dimensional cross-coupling error

E_d	Non-dimensional digital error
T_s	Non-dimensional sampling period
ϕ_f	Freedom in position of a simple limit cycle about a switching line
$\triangleq$	Equal by definition

APPENDIX B

DERIVATION OF ERROR DETECTOR DEAD ZONE REQUIRED TO PREVENT A LIMIT CYCLE WHEN INPUT ACCELERATION IS ZERO

Consider a limit cycle of K time intervals in the positive torque mode followed by L time intervals in the zero torque mode, K time intervals in the negative torque mode and L time intervals in the zero torque mode. The limit cycle must be symmetric in that the sum of the torque pulses over a limit cycle must be zero. Such a limit cycle is shown in Figure B-1. Two requirements must be met in order that the limit cycle can exist:

$$\phi_{(2K+L-1)} - \phi_{(K-1)} \geqq \phi_{\Delta} \tag{B-1}$$

$$\phi_{(K+L-1)} - \phi_{(2K+2L-1)} \leqq \phi_{\Delta} \tag{B-2}$$

Now suppose that switching line 1 is held fixed and switching line 2 is moved to the left until Condition (B-1) cannot be met. Switching will occur prematurely and K will be reduced by one. Since the distance traveled in the torque mode is reduced, the distance traveled in the no torque mode must increase so that Condition (B-1) can be met for the new value of K. Thus L must increase.

Switching line 2 can be moved to the left until K is reduced to one and L is increased to infinity. As L approaches infinity the velocity at N = K + L must approach zero since the pendulum has been damped without being forced for an infinite amount of time. The width of the limit cycle under these conditions can be determined from Equations (2-7) and (2-8):

$$\phi_{(\infty)} - \phi_o = \left[T_s + e^{-T_s} - 1 + 1 - e^{-T_s} \right] \tag{B-3}$$

or

$$\phi_{(\infty)} - \phi_o = T_s \tag{B-4}$$

Now, if switching line 2 is moved to the left such that ϕ_{Δ} is greater than T_s, then K must be reduced to zero and the limit cycle cannot exist.

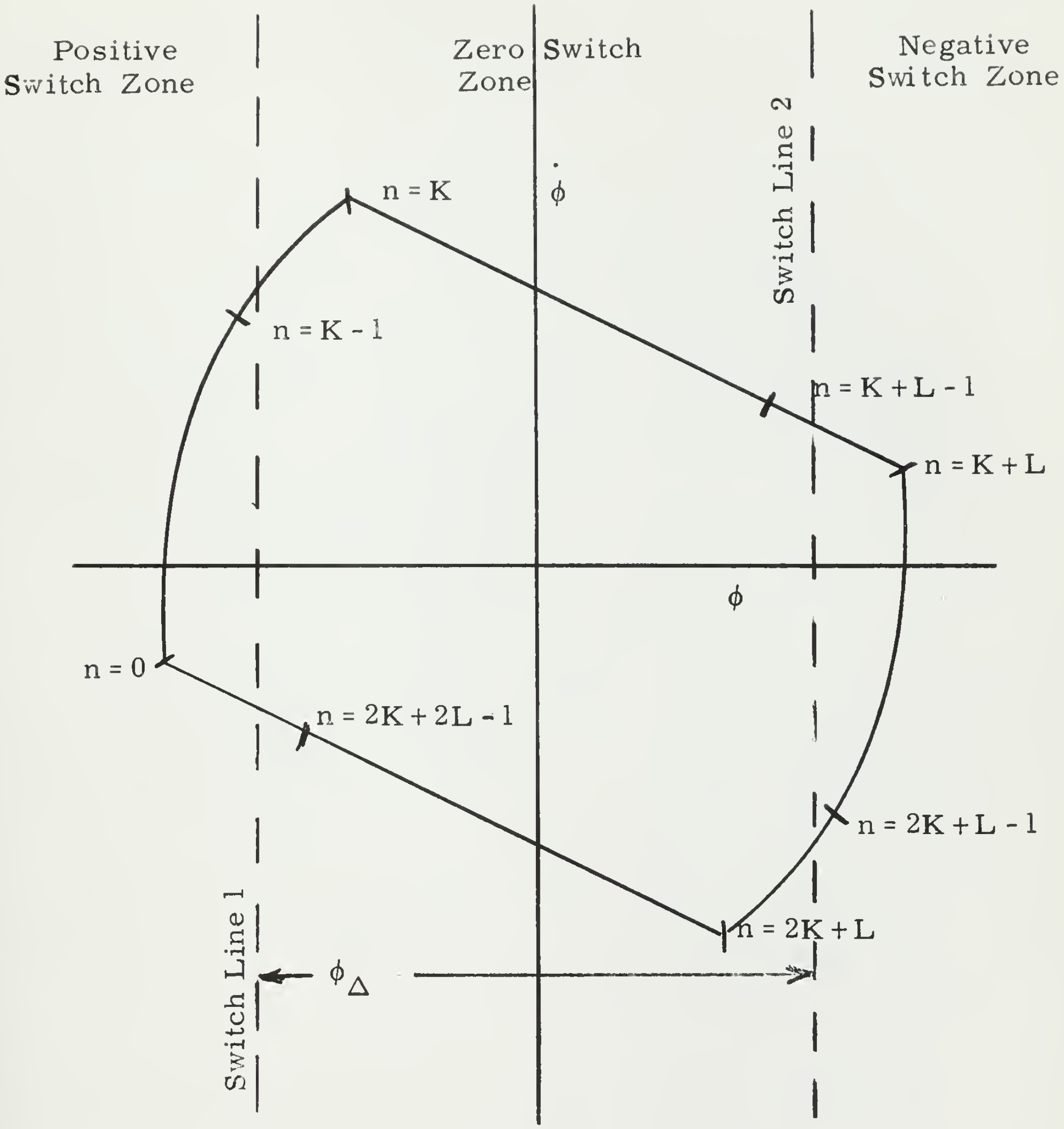


Figure B-1. Limit Cycle With Zero Acceleration.

Thus the condition necessary to prevent a limit cycle is:

$$\phi_{\Delta} > T_s \tag{B-5}$$

APPENDIX C

DERIVATION OF THE FREEDOM IN POSITION OF A SIMPLE LIMIT CYCLE ABOUT THE SWITCHING LINE, SHOWING THE CONDITION NECESSARY FOR A SIMPLE LIMIT CYCLE TO EXIST

A simple limit cycle is defined as a limit cycle which is complete after two switches. For the purposes of this derivation it is assumed that the zero switch zone is of sufficient width to insure that the pendulum in steady state remains in the positive and zero switch zones for positive acceleration and in the negative and zero switch zones for negative acceleration. A typical limit cycle consisting of K sample intervals in a torque mode followed by L sample intervals in the zero mode is shown in Figure C-1.

The freedom in position of the limit cycle is defined as follows:

$$\phi_f \triangleq \phi_{(K+L-1)} - \phi_{(K-1)} \tag{C.1}$$

It is evident from Figure C-1 that the complete limit cycle may be shifted to the right or left over a total range of ϕ_f without disturbing the switching sequence. It is also evident that if ϕ_f is negative, the switching sequence must be altered and the limit cycle described cannot exist.

The velocity summing capability of the system depends upon the relation

$$r = \frac{K}{K + L}\,. \tag{C.2}$$

Equation (C. 2) is without error since the change in null error E_N over a limit cycle is zero. Since K and L are integers, the input acceleration must be the ratio of two integers if the limit cycle is to exist.

The equation of motion of the pendulum is:

$$\ddot{\phi} + \dot{\phi} = (\delta - r) \tag{C.3}$$

where δ is the torquing mode. Equation (C. 3) is linear between torque switches and may be integrated as such with results being valid between switches.

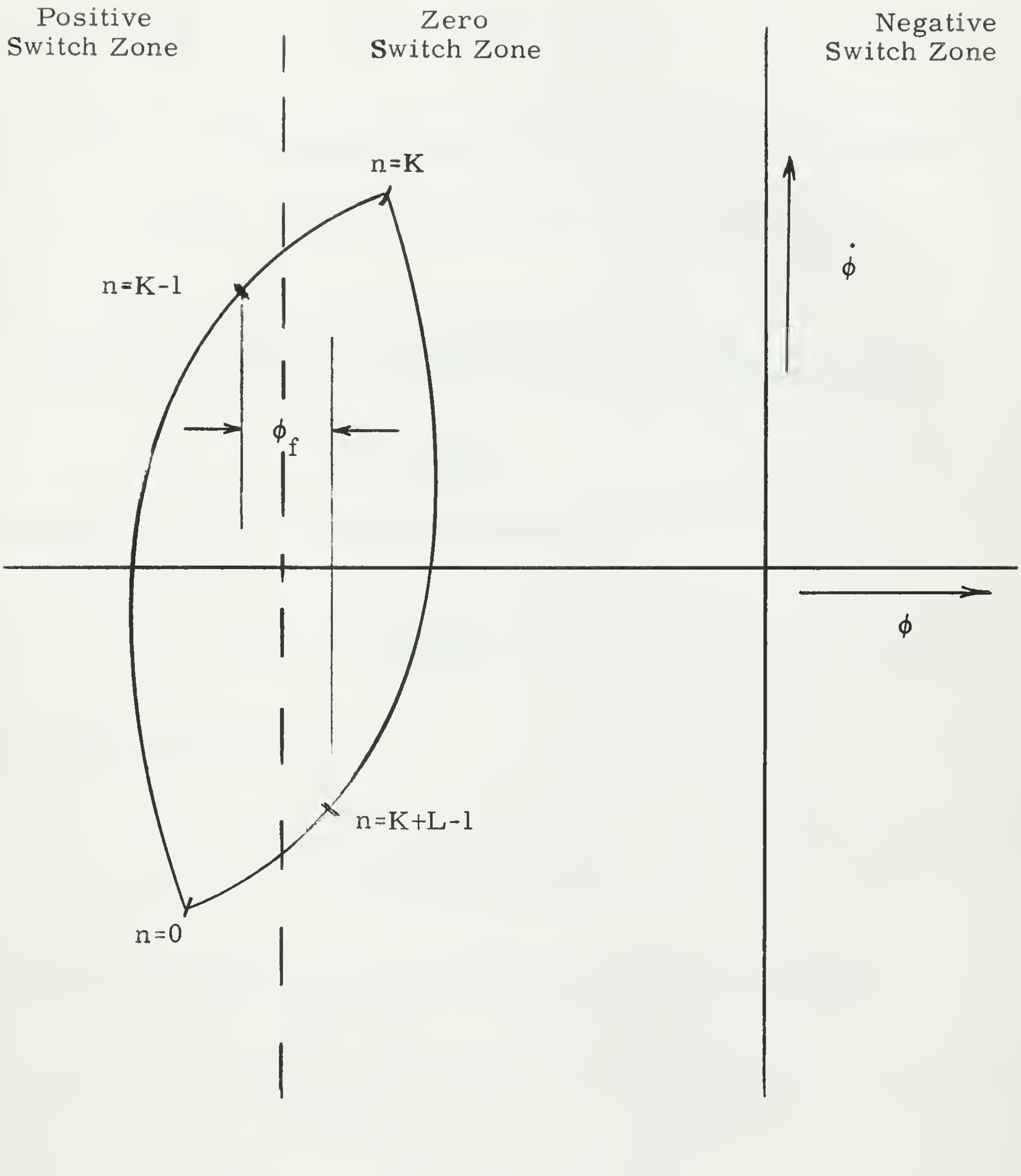


Figure C-1. Simple Limit Cycle.

$$\dot{\phi}_\tau = \dot{\phi}_{\tau_i}\left(e^{-(\tau-\tau_i)}\right) + (\delta - r)\left(1 - e^{-(\tau-\tau_i)}\right) \tag{C.4}$$

$$\phi_\tau = \phi_{\tau_i} + \dot{\phi}_{\tau_i}\left(1 - e^{-(\tau-\tau_i)}\right) + (\delta - r)\left(\tau - \tau_i + e^{-(\tau-\tau_i)} - 1\right) \tag{C.5}$$

A solution for $\dot{\phi}_{(K+L)}$ in terms of $\dot{\phi}_o$ is obtained using Equation (C. 4).

$$\dot{\phi}_{(K+L)} = \dot{\phi}_o e^{-(K+L)T_s} + e^{-LT_s} - e^{-(K+L)T_s} - r\left(1 - e^{-(K+L)T_s}\right) \tag{C.6}$$

where T_s is the non-dimensional sampling interval.

Since a limit cycle is complete after K plus L sample intervals,

$$\dot{\phi}_o = \dot{\phi}_{(K+L)} \tag{C.7}$$

Substituting (C. 7) into (C. 6) and solving for $\dot{\phi}_o$:

$$\dot{\phi}_o = \frac{e^{-LT_s} - 1}{1 - e^{-(K+L)T_s}} + (1 - r) \tag{C.8}$$

Substituting (C. 2) into (C. 8)

$$\dot{\phi}_o = \frac{e^{-LT_s} - 1}{1 - e^{-(K+L)T_s}} + \frac{L}{K + L} \tag{C.9}$$

By using (C. 4), (C. 5), and (C. 9) an appropriate number of times

$$\phi_{(K-1)} = \phi_o + \frac{e^{-LT_s} - e^{-(K+L-1)T_s} - 1 + e^{-(K-1)T_s}}{1 - e^{-(K+L)T_s}} + \frac{LT_s(K - 1)}{K + L} \tag{C.10}$$

and

$$\phi_{(K+L-1)} = \phi_o + \frac{e^{-LT_s} - e^{-(L-1)T_s} - 1 + e^{T_s}}{1 - e^{-(K+L)T_s}} + (1 - e^{T}) + \frac{KT_s}{K + L} \tag{C.11}$$

Substituting (C.10) and (C.11) into (C.1)

$$\phi_f = \frac{2e^{-(K+L-1)T_s} - e^{-(K-1)T_s} - e^{-(L-1)T_s}}{1 - e^{-(K+L)T_s}} + 1 + T_s\left(1 - \frac{KL}{K+L}\right) \qquad \text{(C.12)}$$

Figures C-2 through C-6 show ϕ_f as a function of the non-dimensional sampling period for various values of K and L.

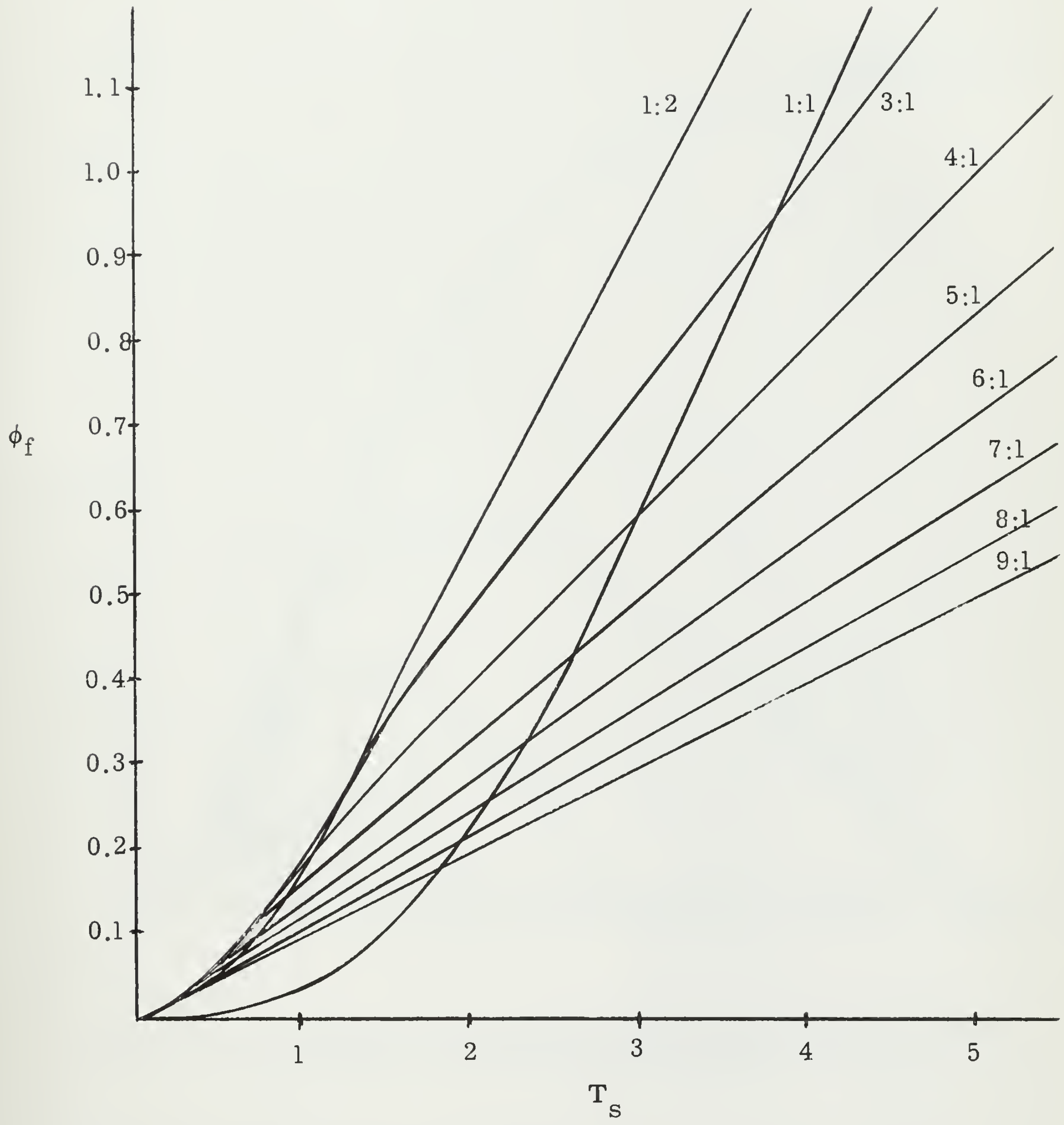


Figure C-2. Freedom in Position of Simple Limit Cycles About a Switching Line; 1:N Limit Cycles.

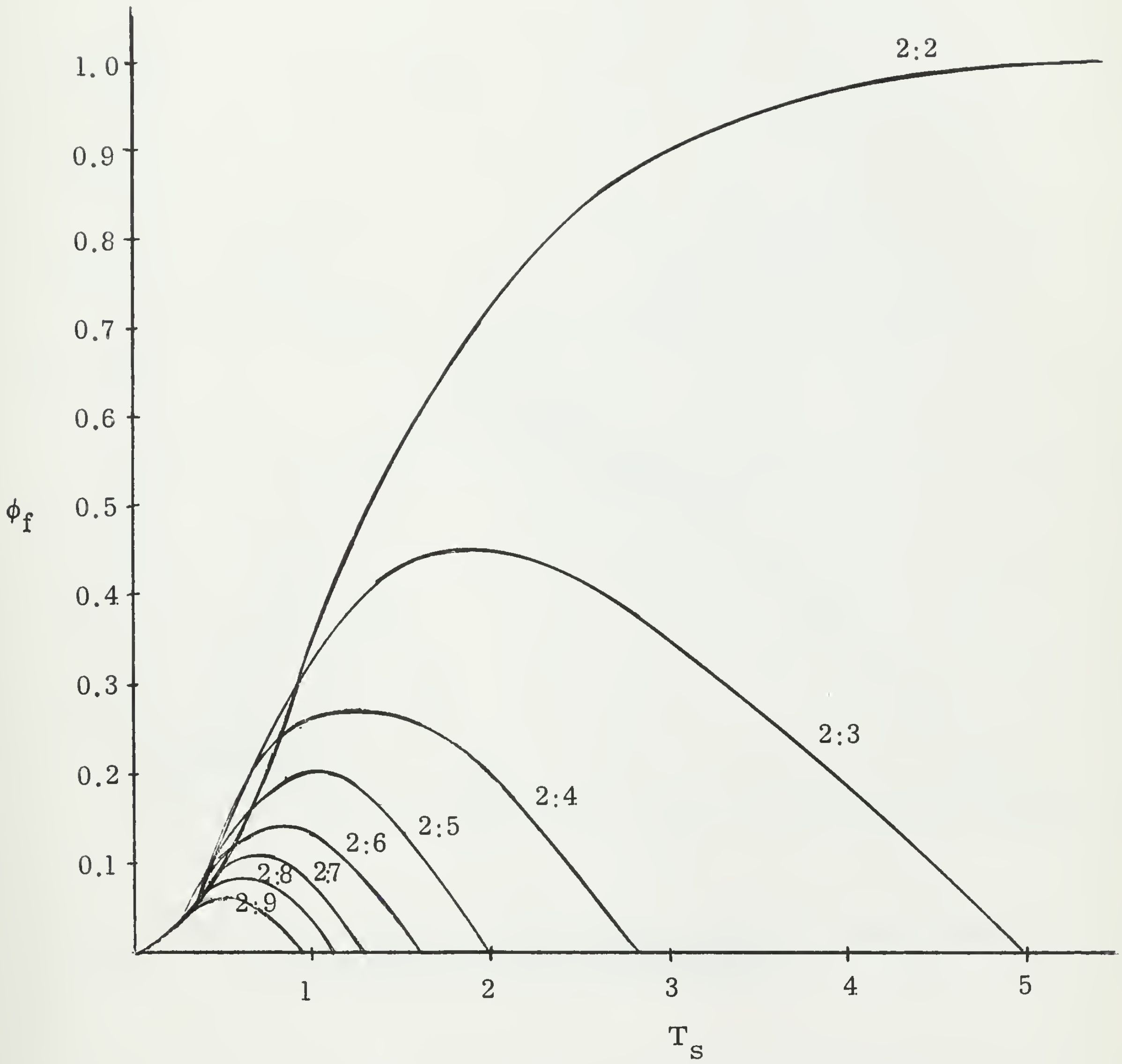


Figure C-3. Freedom in Position of Simple Limit Cycles About a Switching Line; 2:N Limit Cycles.

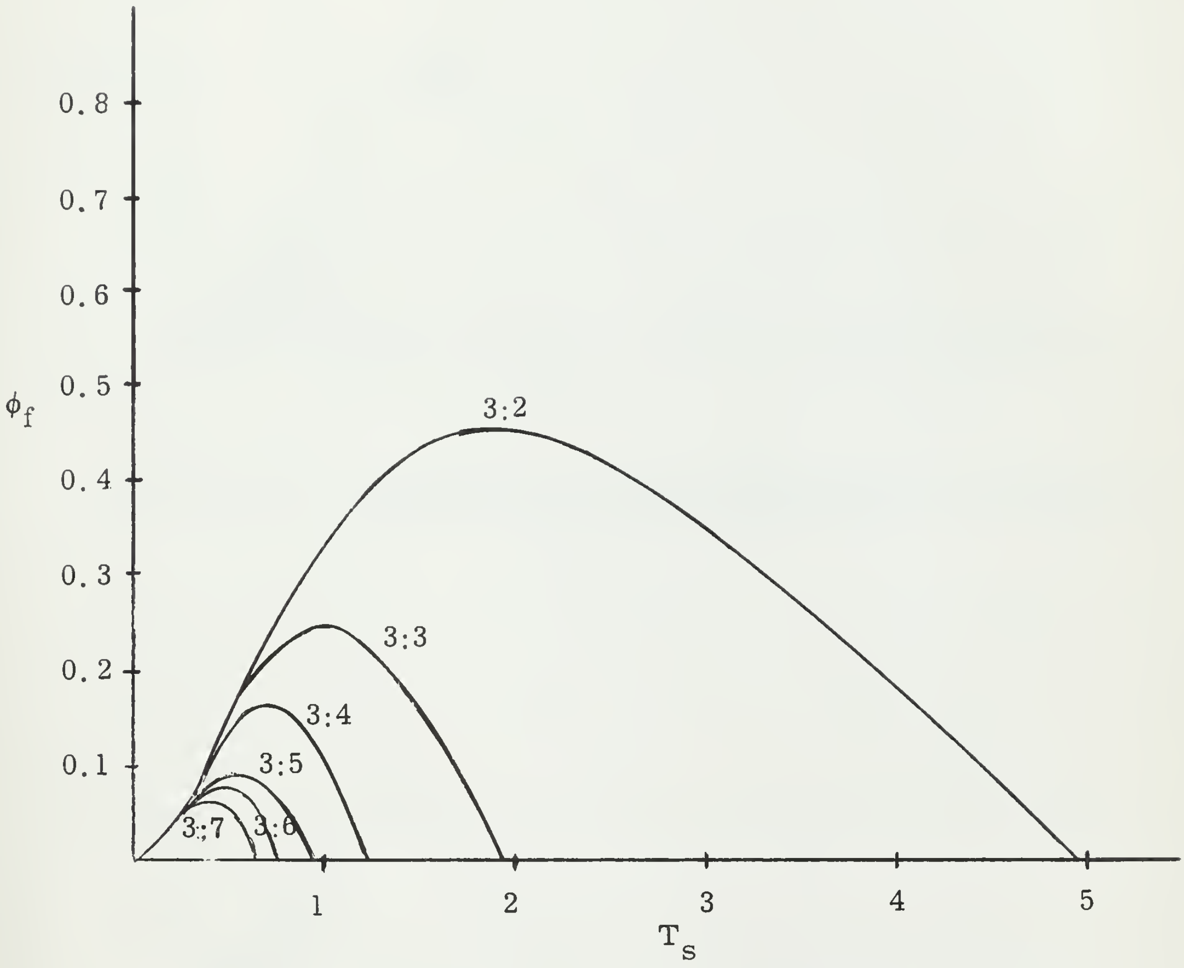


Figure C-4. Freedom in Position of Simple Limit Cycles About a Switching Line; 3:N Limit Cycles.

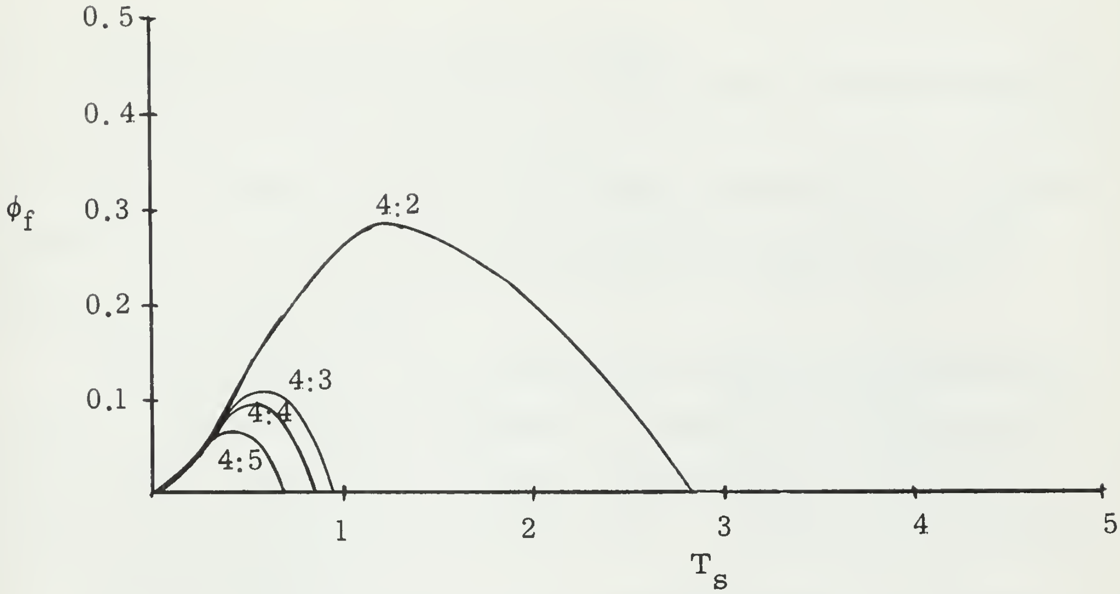


Figure C-5. Freedom in Position of Simple Limit Cycles About a Switching Line; 4:N Limit Cycles.

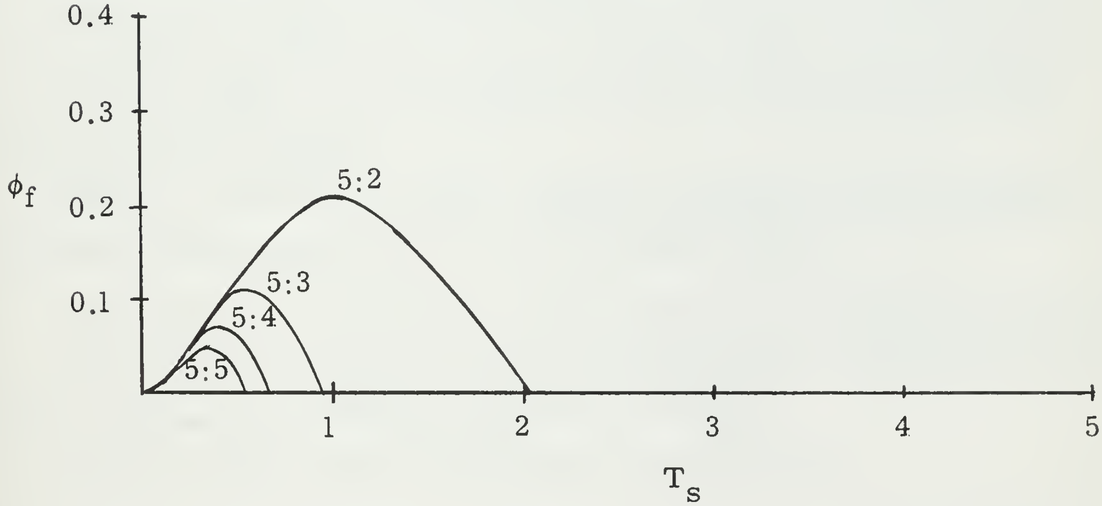


Figure C-6. Freedom in Position of Simple Limit Cycles About a Switching Line; 5:N Limit Cycles.

APPENDIX D

SWITCHING LOGIC DESIGN

D.1. INTRODUCTION AND DEFINITIONS

The object of this appendix is to present the relative complexity of the different switching logics required.

The logic design will be at the logical element level. A logical element is a computer element which performs a nontrivial logic function; that is, some input or combination of inputs produces something new as an output.

In this presentation of switching logic design elementary knowledge of switching design is presumed. Chapter 17 of Reference 6 is an elementary presentation of logical design and was used in developing the switching logic presented here. Switching functions will be in Boolean algebra form.

Figure D-1 defines symbols and characteristics of the logical elements used. The delay repeat is not a standard element. Figure D-2 is an elemental layout of the delay repeat.

Figure D-3 shows the definition of the error level signals. The error detector's output will be zero in the dead zone; but D, the dead zone position signal, can be computed, since it is the absence of the other error level signals.

Table D-1 defines the letter symbols applicable to this appendix only used to define quantities used in the logic design.

D.2. SWITCHING LOGIC FOR ALTERNATING PULSE ZONE COMPENSATION

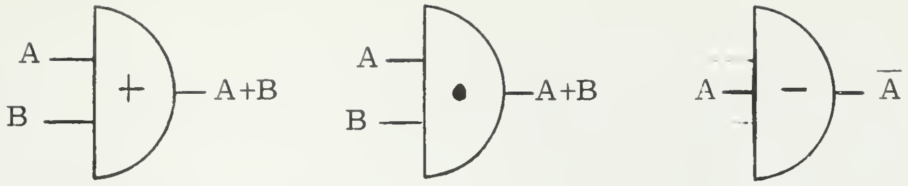
The logic for switching in the alternating pulse zone compensation can be represented by Boolean algebra as follows:

Always torque when in the torque override zones.

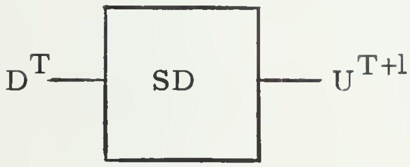
$$P = F \qquad (D.1)$$

$$N = J \qquad (D.2)$$

When in the lead compensation zone, torque one T_s, do not torque next T_s if still in the lead compensation zone, and then repeat the cycle



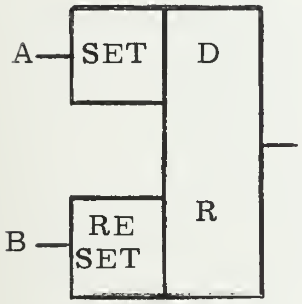
OR   AND   NOT



Single Time
Interval Delay

D^T	U^{T+1}
0	0
1	1

Characteristics
Single Time Interval Delay



C	R	S	U^T	U^{T+1}	D^{T+1}
1	0	1	1	0	1
1	0	1	0	1	1
1	0	0	0	1	1
1	1	0	0	1	0
1	1	0	0	0	0
1	0	0	0	0	0

Storage Characteristics

DELAY REPEAT

GATE

Figure D-1. Symbols and Characteristics of Logical Elements.

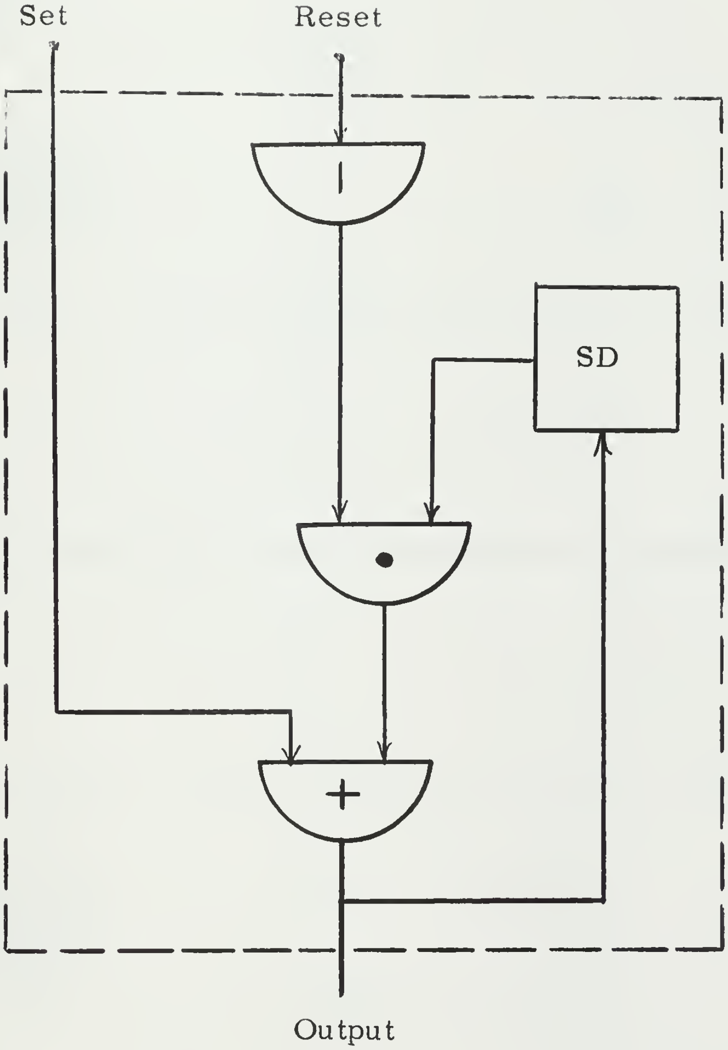


Figure D-2. Elemental Design of the Delay Repeat.

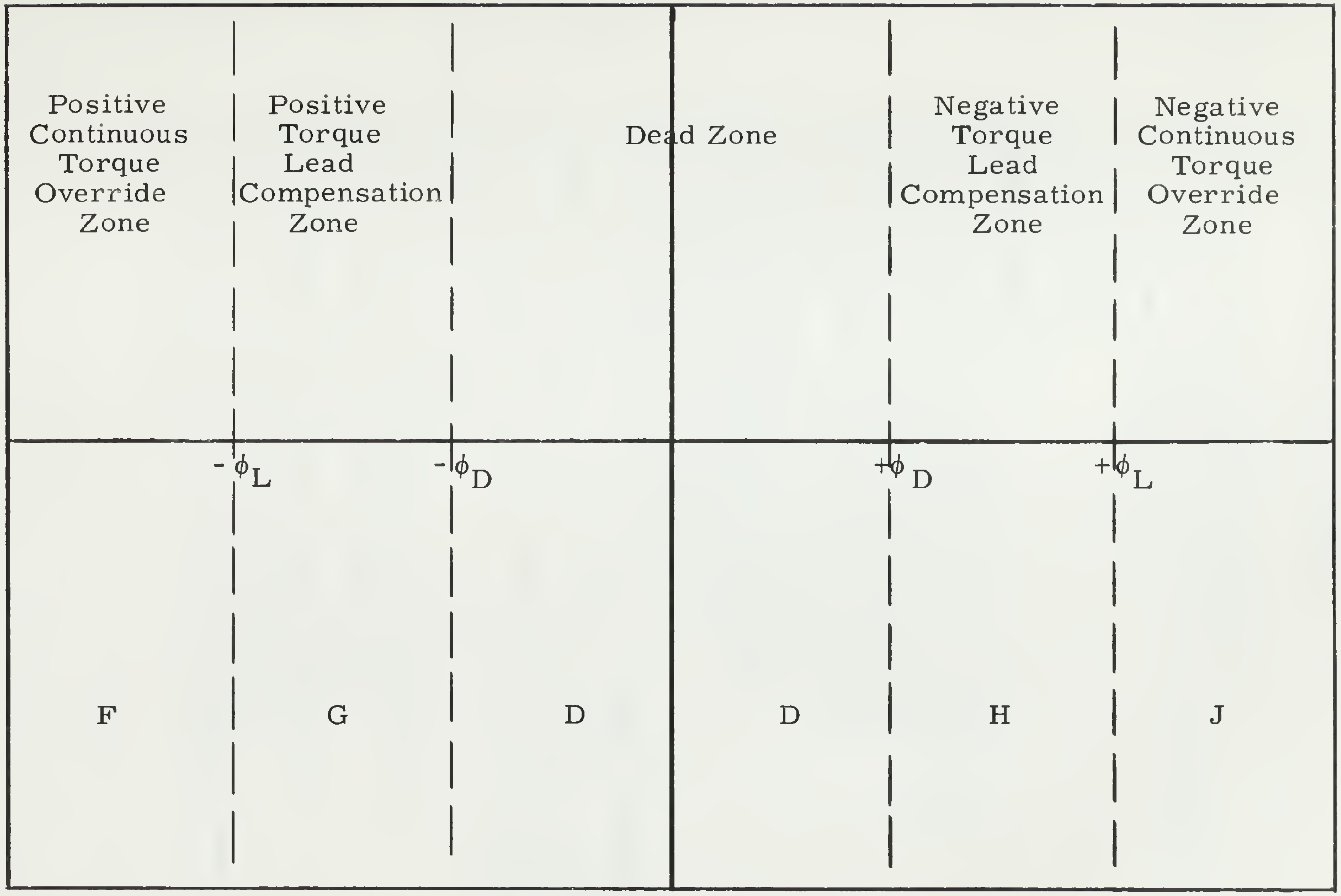


Figure D-3. Phase Plane Location of Error Level Signals Used in Logic Compensation.

TABLE D-I

DEFINITION OF LETTER SYMBOLS FOR SIGNALS USED IN SWITCH LOGIC DERIVATIONS ONLY

A	-	Signal A at present time instant
$\overline{A}$	-	Not A
A^{T+1}	-	Value of A at first preceding interval
A^{T+x}	-	Value A has maintained constant for x preceding periods. x may take any integer value.
P	-	Positive torque
N	-	Negative torque
D~	-	Error in dead zone
F	-	Error in positive continuous pulse override zone
J	-	Error in negative continuous pulse override zone
G	-	Error in positive lead compensation zone
H	-	Error in negative lead compensation zone
K	-	Positive torque pulse as a result of G
L	-	Negative torque pulse as a result of H
Q	-	Intermediate signal in computing K
R	-	Intermediate signal in computing L
(+ or -) $\frac{U}{2}$	-	Mean error bias correction magnitude when switching about ϕ equal (- or +) ϕ_D
(+ or -) V	-	Mean error bias correction magnitude less U/2 when switching about ϕ equal (- or +) ϕ_L
W	-	Negative error sense
X	-	Permit positive torquing for more than one T_s
$\overline{X}$	-	Positive torque for not more than one T_s
Y	-	Permit negative torquing for more than one T_s
$\overline{Y}$	-	Negative torque for not more than one T_s
B	-	Error has been in the dead zone this sampling instant and the one previous

as long as in the lead compensation zone.

$$K = G \cdot \overline{K^{T+1}} \qquad (D.3)$$

$$L = H \cdot \overline{L^{T+1}} \qquad (D.4)$$

Combining

$$P = F + K \qquad (D.4)$$

$$P = F + G \cdot \overline{K^{T+1}} \qquad (D.5)$$

$$N = J + L \qquad (D.6)$$

$$N = J + H \cdot \overline{L^{T+1}} \qquad (D.7)$$

The switching logic described by equations (D. 5) and (D. 7) is shown in Figure D-4.

D. 3. SWITCHING LOGIC DESIGN FOR INVERSE ORDER ALTERNATING PULSE ZONE COMPENSATION

The switching logic for the inverse order alternating pulse zone compensation is like the switching logic for the alternating pulse zone compensation except that when in the lead compensation zone the opposite action of that taken the preceding period, no matter which zone determined the action, is desired. This is done by changing equation (D. 3) to

$$K = G \cdot \overline{P^{T+1}} \qquad (D.8)$$

and equation (D. 4) to

$$L = H \cdot \overline{N^{T+1}} \,.. \qquad (D.9)$$

Then

$$P = F + G \cdot \overline{P^{T+1}} \qquad (D.10)$$

$$N = J + H \cdot \overline{N^{T+1}} \qquad (D.11)$$

The switching logic described by equations (D. 10) and (D. 11) is shown in Figure D-5.

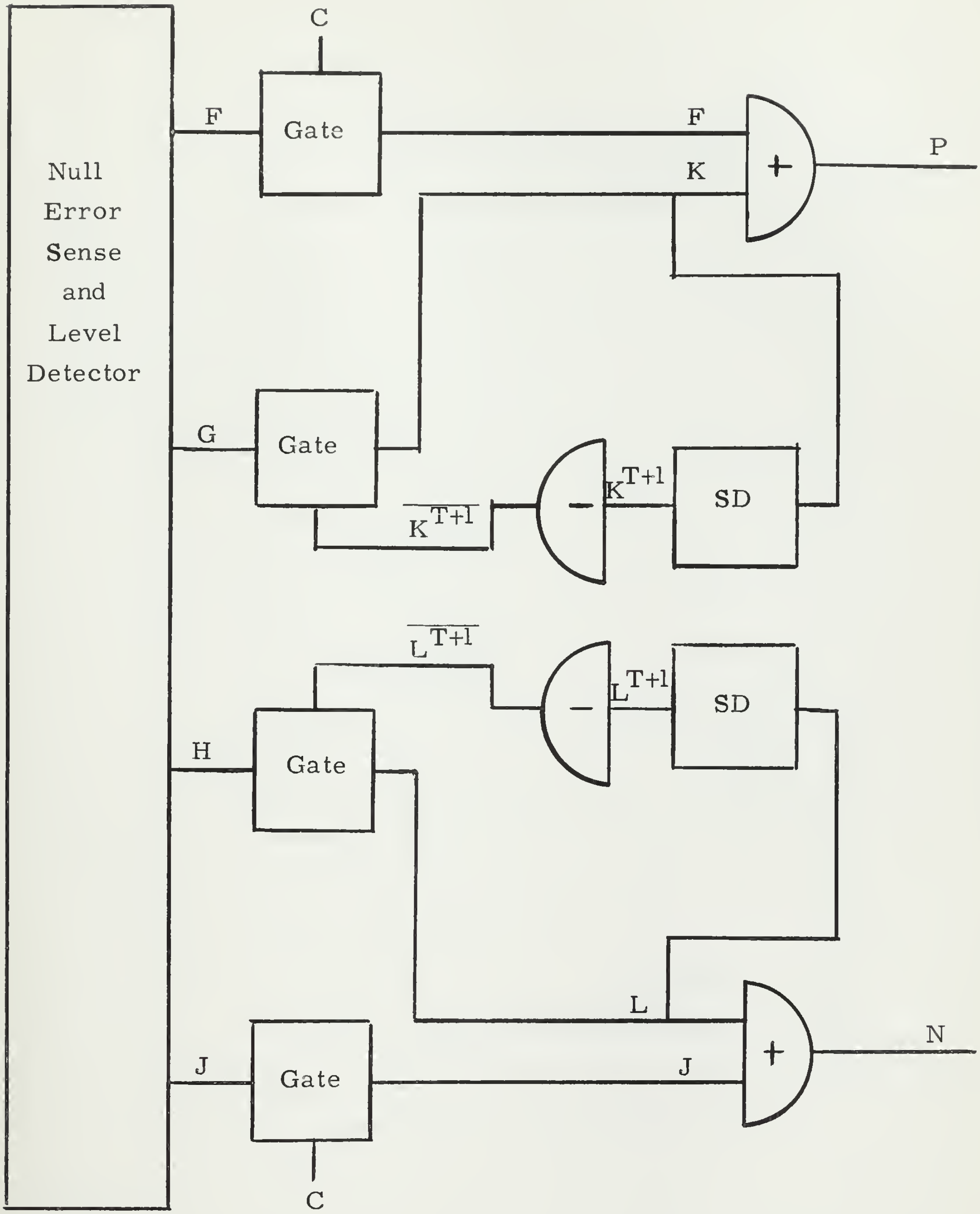


Figure D-4. Switching Logic For Alternating Pulse Zone Compensation.

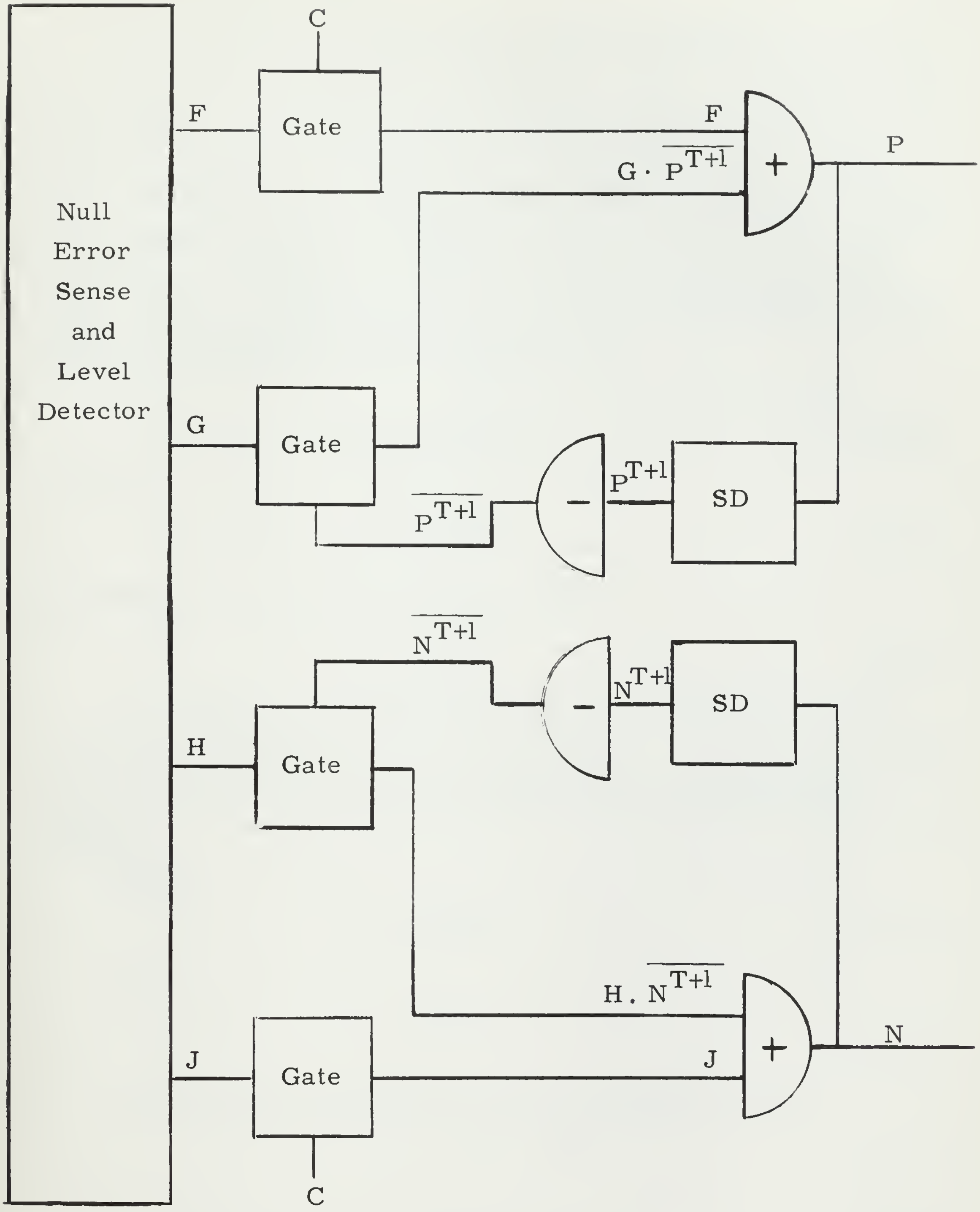


Figure D-5. Switching Logic for Inverse Order Alternating Pulse Zone Compensation.

D.4. LOGICALLY FORCED LOWEST ORDER COMPENSATION SWITCHING LOGIC

In the logically forced lowest order compensation, for values of r less than 0.5 the system is allowed to torque for only one sample period, T_s, at a time, and for values of r greater than 0.5 the system is allowed to not torque only one sample period, T_s, at a time. The logic will be changed from multiple T_s positive torquing, X, to single T_s positive torquing, $\overline{X}$; or from multiple T_s negative torquing, Y, to single T_s negative torquing, $\overline{Y}$; when the error signal remains in the dead zone for two consecutive sample periods. This is represented by Booloean algebra by

$$B = D \cdot D^{T+1} , \tag{D.12}$$

reset X

$$\overline{X} = B^{T+x} , \tag{D.13}$$

and reset Y

$$\overline{Y} = B^{T+x} \tag{D.14}$$

The logic will be changed from single T_s torquing, $\overline{X}$ or $\overline{Y}$, to multiple T_x torquing, X or Y respectively, when the error signal passes the $+\phi_L$ or $-\phi_L$ line respectively in an increasing error direction.

Set X_g

$$X = F^{T+x}. \tag{D.15}$$

Set Y_g

$$Y = J^{T+x} \tag{D.16}$$

The torque switching can be summarized as follows:

$$P = F + G \cdot X + G \cdot \overline{X} \cdot \overline{P}^{T+1} \tag{D.17}$$

$$N = J + H \cdot Y + H \cdot \overline{Y} \cdot \overline{N}^{T+1} \tag{D.18}$$

The logic described by equations (D.12) through (D.18) for the logically forced lowest mode compensation is shown in Figure D-6.

D.5. ERROR BIAS CORRECTION SWITCHING LOGIC

Proceeding from positive error to negative error, the change in error bias correction is +V when changing from $+\phi_L$ to $+\phi_D$, +U when changing from $+\phi_D$ to $-\phi_D$, and +V when changing from $-\phi_D$ to $-\phi_L$. The sign of the change in error bias correction is negative proceeding from negative to positive. The magnitude of U and V may be adjusted to make the best correction for the mean error due to switching about a line.

When it is necessary to correct for the error only when switching about the ϕ_L lines, the Boolean algebra equations for the switching are

$$+\left(V + \frac{U}{2}\right) = F \cdot D^{T+x} + D \cdot L^{T+x} \qquad (D.19)$$

$$-\left(V + \frac{U}{2}\right) = D \cdot F^{T+x} + L \cdot D^{T+x} \qquad (D.20)$$

Only one quantity may be stored at a time. Figure D-7 shows the required switching logic.

If it is necessary to correct only for the error bias in switching about the ϕ_D lines as in the uncompensated case or the logically forced lowest mode case again a special starting condition need not be used unless the pendulum was restrained until counting started. This is because the pendulum will be switching about the initial switch line when counting starts. +U must be switched when entering the F or G zones when -U was last switched. -U must be switched when entering the H or L zones when +U was last switched.

$$+U = (F + G) \cdot (-U)^{T+x} \qquad (D.21)$$

$$-U = (H + L) \cdot (+U)^{T+x} \qquad (D.22)$$

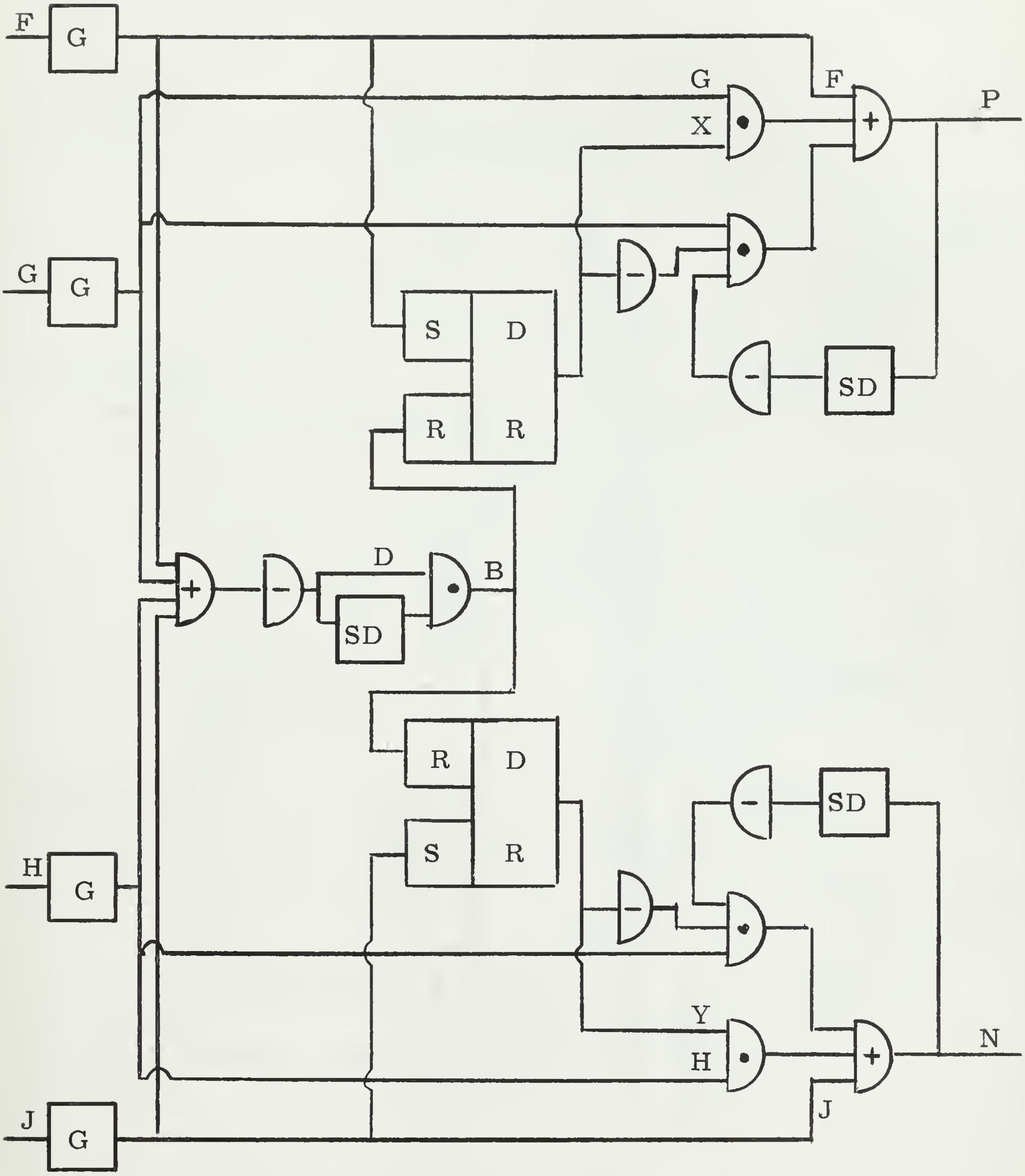


Figure D-6. Torque Switching Logic for Logically Forced Lowest Order.

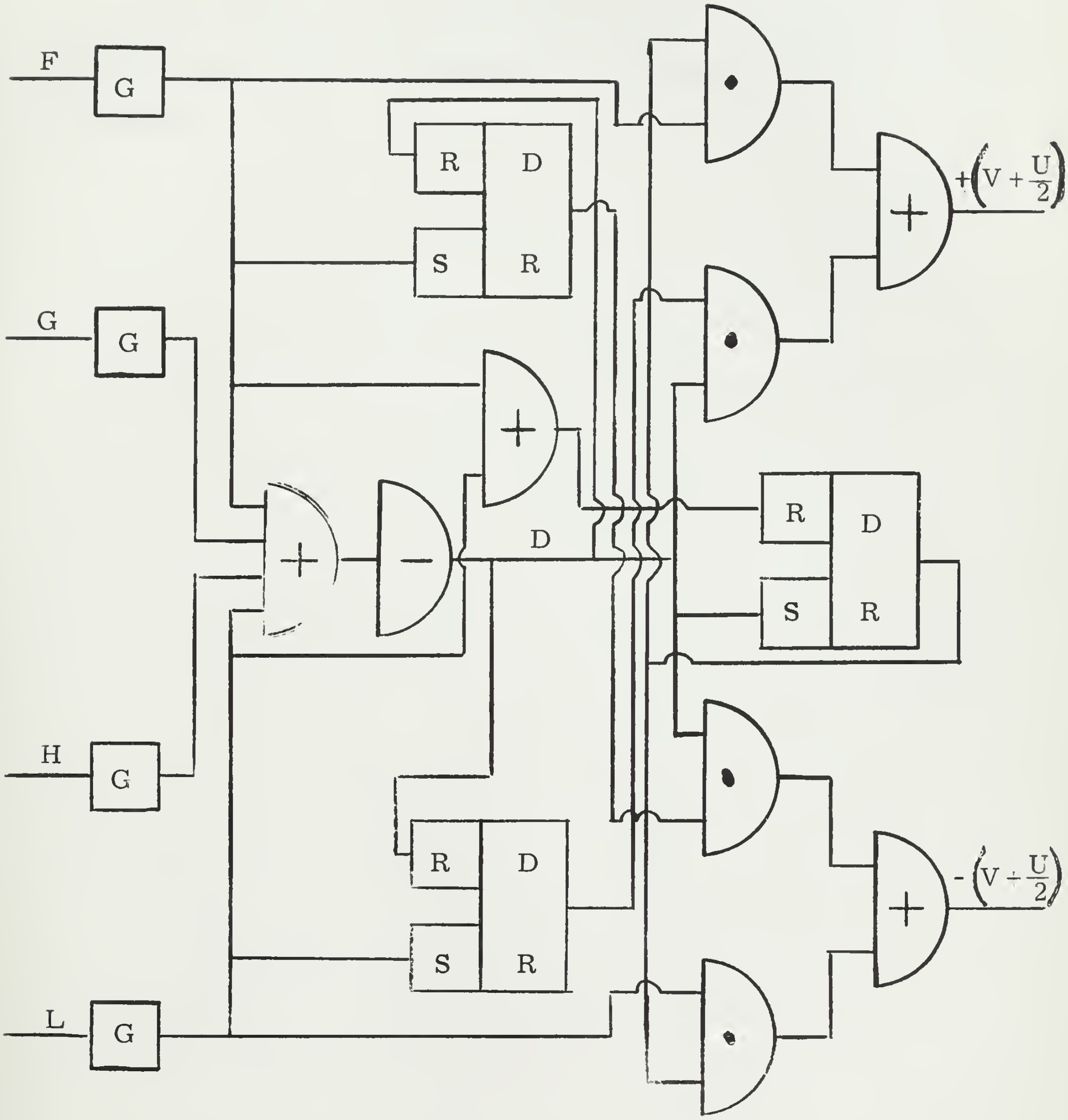


Figure D-7. ϕ_L Switching Line Error Bias Correction Switching Logic.

-U will reset $(+U)^{T+x}$ and +U will reset $(-U)^{T+x}$.

Figure D-8 shows the two state ϕ_D switching line error bias correction switching logic.

A four state error bias correction to correct for the error bias due to switching about each of four possible switching lines can be constructed using a combination of the two preceding error bias corrections.

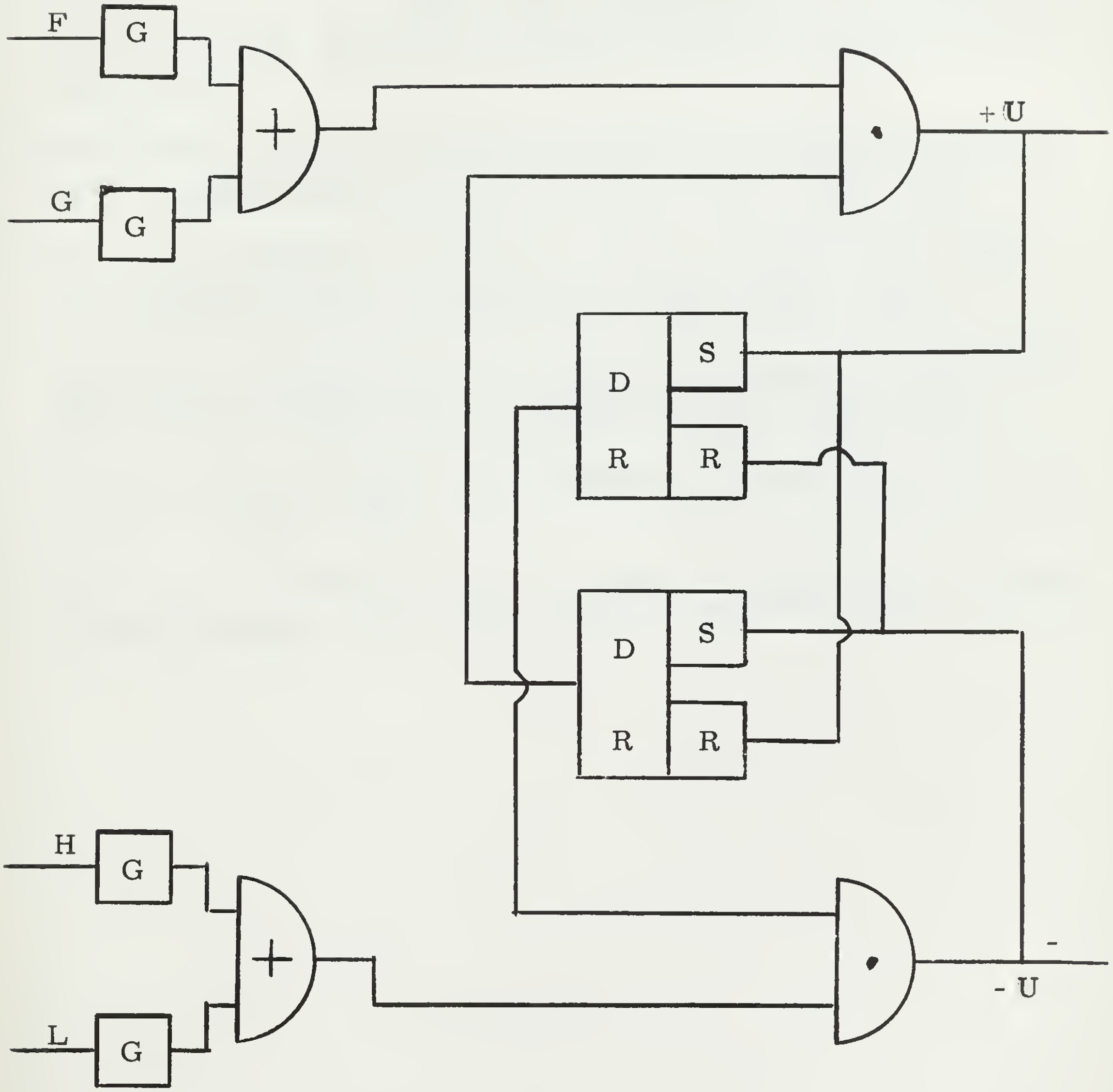


Figure D-8. ϕ_D Switching Line Error Bias Correction Switching Logic.

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